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The Teaching of Mathematics

in the

Higher Schools of Prussia

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The Teaching of Mathematics in the Higher Schools of Prussia

BY

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Preface

The well-known fact that the Prussians have long been studying the problems of education systematically, thoroughly, and successfully, led me recently to spend nearly an entire academic year in examining the outcome of their study as evinced in the present status of the work of education in Prussia. Some of the results of this examination are presented herewith.

It was my primary aim in examining the Prussian higher-school system to ascertain their methods of teaching mathematics, *but the work in mathematics cannot be understood without some acquaintance with the entire system of which the mathematical work is a part and by whose spirit it is dominated. Consequently the following account combines a general sketch of the Prussian higher-school system with a more specific and detailed description of the work in mathematics.*

The reader who desires to learn more of the

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German higher schools in general than is contained in the condensed outline which suffices for the purposes of the present report, is referred to the work of Russell[1] *on the German higher schools.*

The material for the following account has been collected in part from the official and other publications relative to these schools, and in part from many visits of observation to the institutions themselves. The permit requisite for these visits was secured for me from the Minister of Education through the intermediation of His Excellency, the American Ambassador at Berlin, to whom I feel deeply grateful for this kindness. I met with the utmost courtesy in all my visits; the various Directors and instructors were most obliging and did all in their power to assist me in gaining the information of which I was in quest, and I wish to express my most hearty appreciation of their hospitable attentions.

My thanks are also due to Professor H. Maschke,

[1] Russell, *German Higher Schools: The History, Organization and Methods of Secondary Education in Germany.* Longmans, Green & Co., 1898, pp. 455. This work has come to my hands since the following pages were prepared, and I have read it with great interest. It is thorough and scholarly as well as interesting, and I know of no other work in English so complete, satisfactory, and reliable.

of the University of Chicago, formerly Professor in the Luisenstädtische Gymnasium, Berlin; Mr. C. E. Linebarger, Instructor in the Lake View High School, Chicago; and Mr. E. R. Breslich, a graduate of a Prussian gymnasium, for reading the following pages in manuscript, critically from their respective stand-points.

J. W. A. Young.

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THE TEACHING OF MATHEMATICS IN PRUSSIA

I

Introduction

The teaching of mathematics in the higher schools of Prussia deserves the serious attention of those interested in the teaching of mathematics in America, not only because it is the fruit of long labors by a nation that has stood and still stands in the forefront of educational progress, but also because a comparison between the work accomplished in mathematics in Prussia and in the United States reveals a disparity of a character so grave that American educators cannot afford to pass it by unheeded.

Prussian results.

There are very few subjects in which a comparison of the quantity and quality of work accomplished under different curricula and methods can be instituted in any but the most general way. The result is affected by so many elements which cannot be

Possibility of comparison.

specified in black and white, or tested by examinations, that it is wellnigh impossible to find a satisfactory standard of comparison. In mathematics, however, this is the case to a much smaller extent than in other branches. The subject-matter of school mathematics has long since been so systematized, and its nature permits so little variation in the topics taken up and the order of their consideration, that the quantity of work done may be quite clearly described by a list of topics and its quality sufficiently well tested by examinations.

The present study of the Prussian secondary school system was begun under the impression (quite current in America) that, while the work of the Germans is perhaps more thorough, it is accomplished with a greater outlay of time than is devoted to the same subject-matter in this country. But comparison of curricula and time-schedules reveals the startling fact (which will be substantiated in detail in the sequel) that *in the work in mathematics done in the nine years from the age of nine on, we Americans accomplish no more than the Prussians, while we give to this work about seven-fourth* (1.72) *times as large a fraction of the total time of instruction as do the Prussians.*

Ratio of seven to four.

In other words, *the Prussians give about 1.2 years of the nine years in question to mathematics, accomplishing fully as much as the Americans, who*

give about 2.1 years to the same work—a difference of nine-tenths of a year, or one-tenth of the total time of instruction in these nine years.

This state of affairs is certainly one which demands most careful consideration at our hands. Is there really so great a disparity as appears on the face of the time-schedules? Is there in reality any disparity at all? If so, to what is it due? Is it possible for Americans so to modify their system and methods as to diminish the disparity? What lessons can Americans learn from the Prussian system? These and other questions suggest themselves at once, and it is with these questions in the foreground that the following account of the Prussian system should be read.

Questions for consideration.

The writer wishes to say at the outset that, while he believes that there is a real disparity and that it ought to be lessened, he by no means advocates that the Prussian system as such be adopted or imitated here. It does seem, however, that the indisputable superiority which the facts mentioned above show the Prussian system to have in its own environment, over the American system in its environment must oblige American educators to study the Prussian system most carefully, especially along its lines of divergence from their own. While the outcome of such study may be that little or nothing is found which we can directly

What is to be learned.

adopt, hints may perhaps be gleaned which we may *adapt* to our own circumstances with signal profit. Education is more a problem of humanity than of nationality, and while distinctively German methods might not prove strong elsewhere, those results which the Germans have attained as *men* and not as Germans must be of great significance the world over.

II

The General Status of the Higher Schools

The distinction between the *common* and the *higher* schools (*Volks- und höhere Schulen*) must be noticed at the very outset. This is not a distinction in any way analogous to our grades and high schools, but constitutes a complete differentiation of the boys from almost or quite the beginning of their school career into two distinct classes. In the *Volksschulen* the aim is to train good and faithful citizens; the process is called *Erziehung* ("bringing up"). In the higher schools, on the other hand, the aim is to impart learning and to turn out men who are educated or cultured (*gebildet*); the process is called *Unterricht* (instruction) and leads to privileges and responsibilities before the civil and the military law, and the unwritten social law as well. The higher schools proper take the boys at the age of nine and have a curriculum covering nine years; in many cases a preparatory school with a course of three years is connected with the institution, so that a boy of six years may step

Common vs. higher schools.

into the work of a *higher* school. As the common schools do very little in mathematics, only the work of the higher schools will be considered in what follows. Several American writers have already given general descriptions of the German common school system, to which the reader who may be interested in the work of these schools is referred.[1]

A thorough German compend covering the entire educational system is that of Petersilie,[2] in which all grades of institutions from the Universities down are described and their regulations collated. This is done in considerable detail for Germany, and in a summary manner for the other principal countries of Europe.

Education a duty or a privilege.

In Prussia the state regards attendance upon a higher school as a *privilege;* for the common good the state may restrict the number of persons admitted to such attendance. But attendance upon the common schools is regarded as the *duty* of those not having better opportunities, and is enforced by the state. The attendance upon

[1] For example, Klemm, *European Schools*, D. Appleton & Co., 1889; Seeley, *The Common Schools of Germany*, Kellogg & Co., 1896.

[2] Petersilie, *Das öffentliche Unterrichtswesen im deutschen Reiche und in den übrigen europäischen Kulturländern*, Leipzig, 1897, 2 Bde., pp. 448, 608.

higher schools is now being restricted by the state by the simple expedient of founding fewer new institutions than would be adequate to meet the present demand for admittance, which is far in excess of the number that can be received by the institutions now in existence. This is done to abate the crying evil which Bismarck called the *Abiturientenproletariat* ("beggar-graduates"). The graduate of a higher school is admitted to occupations closed to all others, and the social usages are such as to prevent him, on pain of losing caste, from entering any of another large group of occupations. Consequently the occupations which are considered suitable for graduates are terribly overcrowded, and since the pressure cannot be relieved by overflow into other occupations, it must be relieved at the source of supply. The condition of affairs is graphically illustrated by an experience of an American resident of Berlin. An educated German called in silk hat and gloves to *beg*. "Why don't you work?" asked the common-sense American, and was met with the indignant reply, "That would not be in keeping with my social station" ("*Das wäre nicht standesgemäss*").

The higher schools are divided into three types: the *Gymnasium*, with both Latin and Greek; the *Realgymnasium*, with Latin but no Greek; and the *Oberrealschule*, with neither

Latin nor Greek. The characteristics of these institutions will be described later on, this brief mention sufficing for the present. The corresponding work in America is divided between institutions of differing character, and there are here no single institutions analogous to the German higher schools; we shall therefore be obliged to retain the German names in speaking of these schools.

The types of higher schools.

III

The Government

The primary source of educational authority in Prussia is the king. The present king (Emperor William II.) takes an active interest in the work and was himself educated in a gymnasium. The head of the actual educational work is the Minister of Spiritual, Educational, and Medicinal Affairs (*Geistlicher, Unterrichts- und Medizinal-Angelegenheiten*), who is appointed by the king and is a member of the royal cabinet. He is assisted by over twenty active councillors for educational matters (*vortragende Räthe*). The Minister, in turn, appoints a school-board (*Provinzial-Schulkollegium*) for each of the thirteen provinces of Prussia, to whom the detailed supervision of the schools is intrusted.

The governing bodies.

Besides particular communications to the separate institutions, both the Ministry and the provincial boards issue frequent circular orders or general bulletins (*Circularverfügungen*) to the institutions respectively under their charge. These take up on

Circular orders.

occasion matters of pedagogic method, and of administration even in detail, and thus tend to produce great uniformity in the work of all the institutions of the kingdom. The orders of interest to the general public are usually published in the annual announcement (*Programm*) of the school, and in treating of the latter, citations will be made.

All the orders since the beginning of the century which are still in force have been collected in the work of Wiese-Kübler,[1] which is officially recognized, and upon which some of the statements of this paper are based. The orders are well grouped and indexed, and the work as a whole constitutes a complete and authoritative exposition of the organization and regulations of the Prussian higher school system.

The provincial school-board.

The provincial school-board is composed of picked men of experience in school-work, who devote their entire time to the duties of this position and who receive the highest salary paid. Each member has a number of institutions assigned to him for special personal supervision; he keeps in close touch with each, and informed as to its work by personal communication with the Di-

[1] Wiese-Kübler, *Verordnungen und Gesetze für die höhere Schulen in Preussen*, 3te Aufl., Bd. I., 1886, pp. 488; Bd. II., 1888, pp. 521.

rector, by visitations (for inspection of instruction and conferences with the teachers), and by conducting examinations. He is the connecting link between the school-board as a whole and the institutions under his especial charge. The *Schulkollegium* of the province Brandenburg, in which Berlin lies, has commodious and well-fitted quarters (three floors of a large building) and a considerable clerical force.

The head of the institution is the *Director*, appointed by the provincial school-board with royal approval. He administers the affairs of the institution, assigns to each teacher his work, and appoints an "ordinary" (*Ordinarius*) for each class. These are, as it were, subdirectors supervising the work of single classes as the director does that of the entire institution. The ordinary is usually the teacher who gives the largest number of hours of instruction to the class, and consequently the teachers of Latin are sure to be among those who are called on to perform this duty. The ordinary has charge of all the routine supervision of the class as a whole, and comes into closer personal contact with the pupils than do the other teachers.

The school-officers.

Everything not strictly in the usual course of the work of instruction of a class must be referred to the ordinary. Every question not

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strictly in the usual course of administration of a single class must be referred to the director, and so on, each officer referring to a higher authority all questions not falling within the scope of his own well-defined powers.

IV

The Teachers

The first step toward becoming a teacher in the higher schools of Prussia is the acquisition of a liberal education and of sufficient scientific attainments in the subject which the candidate wishes to teach. It is requisite that he have completed the course of a gymnasium (for certain subjects that of a realgymnasium will suffice), and that he have studied three years in German Universities. The adequacy of his preparation is tested by an examination.

Preparation.

This is conducted by a board (*Königliche wissenschaftliche Prüfungscommission*), appointed for that purpose by the Minister of Education. There are ten such boards in the kingdom, one board serving for two provinces in a few cases. Their seat is always in a University town, and their membership is made up almost entirely of University professors.

The examination.

This examination is known as the examination *pro facultate docendi*, or the "Staatsexamen." To be admitted to it the candidate must submit

a. A certificate of maturity (equivalent to our diploma of graduation) from a German gymnasium; if the principal subjects (see below) are taken from the following: Mathematics, natural sciences, foreign modern languages, the candidate may offer a certificate of maturity from a realgymnasium.

b. Documents to show that he has studied three years in a German University (of these at least one and one-half years in a Prussian University); if one of the subjects is English or French, the candidate may by special permission replace one year's University study by study of the language in question in an institution or in a country in which the language is spoken.

The subjects.

The candidate specifies the particular subjects in which he seeks authorization to teach and the grade in which he wishes to obtain the teacher's certificate. There are three grades—lower, middle, and upper—each constituting three years of the nine years' course. He must offer at least two principal and two subordinate subjects. These subjects are selected from a list which is practically that of the subjects taught in the gymnasia (see curricula below), the combinations being subject to a few restrictions which are of little consequence in the present paper.

The scope of the examination is twofold,

testing the candidate's fitness for the post of teacher in general and in particular. The specific requirements are:

1. **General.**—All candidates are examined in philosophy and pedagogy, the German language and literature, and, if Christians, in the contents of Holy Scripture, Church history, and the dogmas of that church (State Church or Roman Catholic) to which they belong. The object of this part of the examination is to determine whether the candidate possesses that general culture which is to be demanded of all instructors in higher schools.

The scope of the examination.

The examination is rigorous, and as it may take up topics from a wide field, and as the examiner in each subject is usually a University professor and always a specialist in the subject, the candidates often anticipate this examination with more apprehension than that in the special subjects which they wish to teach.

2. **Special Attainments.**—To obtain the teacher's certificate (*Oberlehrerzeugniss*), the candidate must obtain the authorization (*Befähigung*) to teach two (principal) subjects in all classes and two (subordinate) subjects in the middle classes. For the subject of mathematics the scope of the examination is as follows:

a. **For Middle Classes.**—Plane and solid geometry, algebra through quadratics, logarithms,

properties of the decimal system of numeration, equations of the third and the fourth degree, spherical trigonometry with applications to mathematical geography, plane analytic geometry, and the elements of the differential and integral calculus.

b. **For Upper Classes.**—In addition to the foregoing the candidate must show that he possesses such acquaintance with the most important branches of higher geometry, analysis, and analytic mechanics as will enable him to treat independently a not too difficult problem from one of these fields, and he must be acquainted with the more important literature of these subjects.

The examination consists of two parts: the written examination, in which the candidate prepares papers privately on assigned topics, and the oral examination, in which he appears before the Commission in person and may be examined at will by each member.

The mode of examination.

1. **The Written Examination.**—Subjects are assigned to the candidate, one from each of his principal subjects, also one from any subordinate subject in which he may wish to obtain the authorization to teach in all classes, and one from philosophy or pedagogy. Not more than three subjects may be assigned altogether. The candidate prepares at home a paper on each

subject assigned. Those from classical philology are treated in Latin, those from foreign modern languages in the language concerned; all others in German, except by special permission.

Eight weeks' time is allowed for the preparation of each paper (an additional eight weeks may be granted upon due application, and still more if necessary), and at the close of the total time accruing for all subjects all the papers are handed in, with the candidate's assurance that they were prepared by himself with no other assistance from persons or books than that specified in detail by him.

If the candidate submits a printed paper written by himself, it may be accepted provided the subject and contents are satisfactory to the board. If the paper have been approved by the faculty of a Prussian University as a dissertation for the Doctor's degree, only the subject of the paper is scrutinized by the examining board.

2. **The Oral Examination.**—If the results of the written examination have been sufficiently good, the candidate is notified to appear for the oral examination which extends over all the subjects offered by him, and includes the general culture topics as well.

Results of the Examination.—The candidate may be passed unconditionally, conditionally,

or rejected. In appropriate cases there are open to him repetition of the examination, supplementary examinations (to work off conditions), and additional examinations (in new subjects for extension of teaching privileges).

The seminary year.

After it has been ascertained by the examination that the candidate is possessed of liberal culture and of sufficient specific scientific attainments in the subjects he wishes to teach, he must next devote a year (*Seminarjahr*) to the study of the art of teaching with a view to the practical exercise of his profession.

To give opportunity for this study, pedagogic seminaries have recently been organized in connection with various ones of the schools, and to these the candidates are assigned in numbers not to exceed six for each seminary.

The director of the institution conducts the work of the seminary, and the seminaries have been located in institutions whose directors are men eminent in the pedagogic world. The work of the seminarist is of three sorts:

1. Weekly conferences of the seminary.
2. Observation of teaching.
3. Teaching under supervision and guidance.

The weekly conference is held under the presidency of the director, and an experienced

professor of the subject which is at the time under special consideration also attends the session and participates in its work. The exercises are varied at the discretion of the director and include informal talks and lectures by him, papers by the members on assigned topics, reports by the members on the instruction they have witnessed; in the case of members of the faculty, this report is confined to a statement of the facts observed, while in the case of colleagues in the seminary, the method of teaching may also be discussed and suggestions for improvement made. The exercises are not very formal, and questions and discussions may constitute an important feature of the proceedings. The seminarists are also guided in the reading of pedagogic literature.

The weekly conference.

The aim is to shape the work of the seminary so as to cover the most important topics during the course of the year; such as the constitution of the higher school system, the curricula of the schools, the aim of the work as a whole and of its various parts, the methods of teaching in general, more detailed consideration of the teaching of those subjects which the members of the seminary expect to teach, the subject-matter of these latter subjects from the teacher's stand-point, text-books and other aids and appliances, etc.

Immediately on entering upon his work, the seminarist is set at observing the instruction which is being imparted throughout the institution. About twelve hours per week are given to these visits of observation which at first range over all subjects and from the lowest to the highest class, in order that the seminarist may understand the scope of the work as a whole and the interrelation of its parts. Later, his visits are concentrated more upon classes in the subjects which he is preparing to teach, and finally upon a class which is soon to be put under his own instruction for a time. As already mentioned, he presents a report of his observations to the weekly conference, and his report receives the criticism of his colleagues and of the experienced teachers who may be present.

Observation of instruction.

The seminarist also attends the regular faculty meetings, but he has no voice in the discussions except when called upon to report concerning pupils under his charge.

When the seminarist has thus visited a designated class for a sufficient length of time to become familiar with the character and methods of the work being done, he is permitted to give the instruction himself under the direction and supervision of the permanent teacher of the class. The entire responsibility for the work done still rests upon

Actual instruction.

the latter, who is usually present during the hour, though less frequently as the candidate progresses satisfactorily. The director and other candidates also often look on. On occasion, the teachers present correct the candidate in the class-room, point out defects or suggest better methods, and he is supposed to take counsel with them privately as to his work. The other seminarists report on his teaching and criticise it freely at the weekly conference, and the teachers who saw him at work add such remarks as they may deem wise. The candidate teaches only a few weeks in any one class, when he is assigned to similar work in another class, his assignments being at first to lower classes, and later to the higher classes.

In the seminary year the candidate is entitled to no remuneration, though there are some small stipends.

At the close of the year the director submits to the provincial school-board a full report on the work of the seminaries; if the work of any has been unsatisfactory, he is debarred from the opportunity of proceeding farther in the course of preparation for the profession of teaching, while the others are advanced by the board to the *trial year*.

This year (*Probejahr*) is usually passed in a different institution from that in which the seminary year was spent, and in it the *candidatus*

probandus is given six to eight hours per week of instruction to do, and the classes are placed under his charge as their regular instructor, though he still works under the supervision and guidance of his superiors. In some cases the exigencies of instruction make it necessary to assign him more instruction to give than that mentioned above. He is paid by the hour for what teaching he does. Prior to the institution of the seminary year (1890), the trial year included work of the character now done in the seminary year.

The trial year.

We have thus seen that, apart from the time requisite to pass the extended and searching examination *pro facultate docendi*, a minimum of five years of special preparation is required of everyone who would become eligible to appointment as teacher in a Prussian higher school. Three of these years are devoted to preparation in subject-matter and two to learning the art of teaching; in the first of the latter, theoretic study of pedagogic problems and methods preponderates, while in the second the candidate tentatively begins the practice of independent teaching. There is small wonder that few of those who survive all of these tests prove later to be poor teachers, and that, on the whole, German teaching leads the world.

Summary of preparation.

Having passed through the seminary year

and the trial year satisfactorily, the candidate is eligible to appointment as teacher. This may be either provisionally as assistant teacher (*wissenschaftlicher Hülfslehrer*), subject to dismissal upon three months' notice, or definitely as instructor (*Oberlehrer*), removable only for serious offences and after formal trial.[1]

The appointment.

In the interests of the work instructors may be

a. Transferred to other positions of not lower rank and not less pay, with payment of moving expenses;

b. Placed temporarily upon the inactive list with prescribed pay; or

c. Placed permanently upon the retired list (*Ruhestand*), with corresponding pension (see below).

The Minister grants the title *Professor* to instructors who have evidenced scientific or pedagogic excellence, and upon the same grounds professors may receive the *rank* of Councillors of the Fourth Class (*Räthe vierter Klasse*). This gives them social equality with the Councillors of the Fourth Class in the other branches of the government, with men of

Ranks and honors.

[1] In certain urgent cases instructors are suspended or forbidden to exercise their functions. They receive half pay while suspended, and the other half is also paid to them if upon trial they are found innocent.

high rank in other professions, who are Councillors of the Fourth Class, such as University professors, and the like, and gives them social precedence of such as have not this distinction. The conferring of these titles has no effect upon the salary of the recipient.

The appointed teacher takes a prescribed oath of office by which he becomes an official of the state. As such he must cultivate loyalty to the king and the realm, advance the interests of education and of his institution as far as he can, and in particular give instruction not to exceed the following number of hours per week:

Duties.

Director, 14–16.

Instructor, 20–22.

The "hours" are at most fifty minutes, and at the close of each hour there is a pause of at least ten minutes. The maximum number of hours may be required only if the classes are small and if the subject of instruction does not demand from the teacher a time-consuming correction of papers.

The teacher also is to do without extra pay such emergency teaching as may be rendered necessary by the death of a teacher of the institution and also to replace free of charge such of his colleagues as may be absent through illness, through leave of absence on the ground of ill-health, through being called into military

service, or through jury duty. If a teacher obtains leave of absence for any cause or other than one of these mentioned, he must himself defray the cost of filling his place during his absence. If a teacher is obliged to travel for his health, to visit springs or baths, an appropriation in addition to his regular salary may be granted him in view of the extra expense to which he is subject.

The salary of an instructor is $648 per annum (1 Mark = 24 cents) during the first three years, at the close of which time he receives an increase of $72 per annum, and each three years thereafter a like increase is made until in twenty-four years the maximum salary of $1,224 is reached. One-half of the instructors in the complete institutions (see below) and one-fourth of those in the incomplete institutions receive annually a fixed addition to their salary (*feste Zulage*) of $216. This addition when once attained is permanent and is usually granted new recipients in order of seniority as vacancies occur (by death or transfer to the inactive or retired list). Instructors also receive an appropriation for rent varying with the population of the town in which the institution is located. This appropriation ranges from $216 for Berlin to $86 in the smallest villages. This is the only item in which the incomes of instructors vary according to the

The salaries.

location of their institutions. Consequently the incomes of instructors will gradually increase in Berlin from $864 to $1,656, and in the smallest villages from $734 to $1,526, while other cities and towns have ranges intermediate between these.

The pensions.

After ten years of service instructors are entitled to pension in case they are permanently disqualified for teaching by physical or mental weakness or by incapacity for the work. The amount of the pension depends upon the number of years which the teacher has served, and also upon the income which he is receiving at the time of retirement. The entire income of the instructor at the time (salary, rent allowance, and fixed addition, if he is the recipient thereof) is the base upon which the pension is computed. After ten years of service the pension is one-fourth of the base, and for each additional year of service one-sixtieth is added to this fraction, until after forty years of service the maximum pension of three-fourths of the base may be received. In counting years of service the seminary year, the trial year, and military service performed after the age of twenty are included. Instructors are not required to contribute to a pension fund.

In Berlin the amount of pension would accordingly range from about $234 at the end of

ten years' service to $1,227 at the end of forty years of service.

After forty years' service instructors are privileged to retire upon pension even though not incapacitated, but this is rarely done. In the programme of one institution a long list of instructors was published who have taught in that institution up to their "Jubilee" (fifty years of service). Each of the older institutions could doubtless furnish a more or less extended list of this kind. The completion of fifty years of service is usually celebrated appropriately. The veteran "*Jubilar*" often receives letters and tokens of distinction from the government, addresses in recognition of his long and valued services from his colleagues, and tributes of respect from his pupils, present and past.

The "jubilee."

The deference paid to age and experience in the German school-world is very marked. The responsible posts, the directorships, the portions of the work of instruction considered the more important or desirable, are usually allotted to elderly or old men. The younger men work with them in cheerful subordination and with genuine respect for the greater wisdom of greater experience.

If we wish to consider the total professional income of the Prussian teacher, we must take the assurance of pension into account. Pensions

can be purchased in life insurance companies in Berlin, by the payment of a fixed sum annually for a term of years, and the study of their rates may enable us to form an estimate of the market value in cash of the assurance of pension which the Prussian teacher enjoys. The conditions on which the pensions are paid by the companies differ somewhat from those of the schools as outlined above, otherwise the rates of the companies might be taken as being the cash value of the assurance of pension. The following table is constructed from the prospectus of a strong company doing business of this sort.

Total professional income.

If payments are begun at the age of twenty-five, the annual payment of P dollars until the age A is attained would secure at the age A a lifelong pension equal to that which the Prussian teacher would receive if retired upon pension at the age A.

A.....	35	40	45	50	55	60	65
P.....	$366	$300	$243	$242	$197	$142	$94

NOTES

1. Different teachers retiring at the same age A might not receive precisely the same amount of pension owing to difference in the ages at which they began service as teachers and the age at which they attained the fixed addition. The range of variation would not be very considerable; in the above table an estimate of the average has been used.

2. In making the table it was assumed that the fixed addition is

attained between the ages of forty-five and fifty years. The teacher's pension at the latter age would thus be quite considerably larger than at the former, which accounts for the very slight decrease in the amount of the annual payment P, requisite for the purchase of the same pensions to take effect at these two ages.

The conditions of assurance correspond quite closely with those to which the teachers' pensions are subject, with one important exception —*viz.*, the payment of pension by the company begins at a stipulated date, neither sooner nor later, and the state of health of the assured or his need of the pension has no influence whatever on his receipt of it. The annual payment of $351 for a period of ten years assures definitely that payment of pensions will be made from the age of thirty-five on, while the teacher receives pension at thirty-five only if permanently incapacitated for work. Likewise the annual payment to the company of $197, for instance, assures the payment of pension at the age of fifty-five, *but not before*, no matter how urgently it may be needed. The teacher, on the other hand, has the *continual guarantee that he will receive pension whenever he may need it.*

It is not easy to deduce from the table above the annual payment which should be made to secure precisely the same guarantees which the school pension system gives. A very rough approximation may perhaps be obtained by regarding the average of all the rates—*viz.*, $224—

as expressing the amount of the mean annual payment sought. Assuming this as the average annual cash value of the pension guarantees, the incomes of the Prussian teachers when compared with those of non-pensioned teachers should be considered as ranging in gradual ascent, sure to each teacher, from $1,088 to $1,880.

Purchasing power in America.

To compare these salaries with American salaries, the difference in the purchasing power of money in the two countries must be taken into account. Conservative estimates made by those who have lived for extended periods of time as settled residents in each country regard the purchasing power of money in Germany as about four-third times that in America. If this be correct, the German incomes would be equivalent in value under American circumstances to a range from $1,451 to $2,507 per annum.

The American work in mathematics corresponding to that for which these salaries are paid in Germany is done in the grades below the high school (five years), in the high school (three years), and in the freshman year in college, the younger teachers receiving the smaller salaries and doing the work of the earlier years.

The above is a crude attempt to estimate the value of the pension guarantees, but by far the

most beneficial effects of the Prussian pension and salary system are not capable of inclusion in a cash estimate. The Germans appreciate that the teacher can do his best only in an atmosphere of financial and mental tranquillity. He must himself be continually growing, and if he is embarrassed by financial cares and harassed by struggles to improve his material position, his growth is retarded and the quality of his work inevitably deteriorates. The teacher is spurred on to his highest achievements by devotion to his calling and by the inspiration of seeking, finding, and imparting truth, but not by the competition of the mart or by the goad of necessity.

Tranquillity of life.

The educational system of Prussia recognizes this truth, and while insisting on high standards and severe tests at the outset, assures a tranquil career to those who have given evidence of their fitness.

The German teacher works with a *sense of security;* security in his position without regard to the occurrences of politics or the whims of the powerful and the influential; security in a modest competency while at work; security in case of a "rainy day;" security in his profession as a lifework, in the assurance that if, as a young man, he maps out for himself a programme of work, study, and research requiring decades for its completion, nothing but the flag-

ging of his own assiduity or the collapse of his physical or mental powers will prevent its successful execution; security, finally, that after a quiet life of patient, undistracted, fruitful toil in his noble vocation he will reach a well-cared for and honored old age, with all needed repose from his well-finished labors.

V

The Pupils

Boys who apply for admission must have completed their ninth year, must be able to read German in both German and Latin characters, have a clear and legible handwriting both in German and Latin script, be able to write from dictation without bad orthographic errors, know a little biblical history, and be familiar with the four fundamental operations on whole numbers. **Admission.**

The pupils are classified into nine classes, each with a course of one year. The following table gives the names of these classes, the abbreviations which we shall use for them, and the average age of the boys in each near the close of the year. In forming this table, the writer examined statistics as to the average age of pupils in twenty-three institutions. These institutions were selected so that all kinds were included and different locations represented. The variations in the average ages as between institutions were found to **Classification and age.**

be so slight that these twenty-three institutions were regarded as giving a correct idea of the range of ages. The average age for each class is published by every institution. The average of these averages constitutes the average of the table. The highest and the lowest average found for each class is also given:

Class.	Oberprima.	Unterprima.	Obersecunda.	Untersecunda.	Obertertia.	Untertertia.	Quarta.	Quinta.	Sexta.
Abbreviation...	IA.	IB.	IIA.	IIB.	IIIA.	IIIB.	IV.	V.	VI.
Average age...	19.4	18.4	17.4	16.5	15.4	14.3	13.2	11.8	10.7
Highest average age.....	20.9	19.4	18.6	17.2	15.9	15.1	13.8	12.7	11.3
Lowest average age.....	18.2	17.5	16.3	15.8	14.5	13.5	12.6	11.5	10.2
Lowest age possible for any single pupil, about..	17.9	16.9	15.9	14.9	13.9	12.9	11.9	10.9	9.9

The table shows that the variation in age among the pupils of any one class cannot be very great, and the appearance of the classes themselves confirms this.

The class-names given to the pupils are formed by adding *ner* to the names of the classes: as *Primaner, Sextaner.*

The teachers of each class, in conference, name the *first boy* or *Primus* of the class. The appointment is made on the basis of scholarship

and is considered a high honor. The *Primus* acts as monitor, supervises the room in the absence of the teacher, keeps the class record, and says prayer at the beginning and the end of the day's session.

VI

The Institutions

The institutions are divided, according to the character of the work and the length of the course, into the following six classes (the customary abbreviations being given in parentheses):

Classification.

A. Nine years' course.
   Gymnasium (G), has both Latin and Greek.
   Realgymnasium (RG), has Latin but no Greek.
   Oberrealschule (ORS), has neither Latin nor Greek.

B. Six years' course.
   Progymnasium (PG).
   Realprogymnasium (RPG).
   Realschule (RSch).

The institutions under B do precisely the first six years' work of the corresponding institutions with nine years' course.

The institutions of Class A are called "Complete Institutions" (*Vollanstalten*), and those of

Class B "Incomplete Institutions" (*Nichtvollanstalten*).

The number of institutions of each kind in Prussia was in 1896 as follows:

Gymnasien	273
Realgymnasien	86
Oberrealschulen	24
Progymnasien	45
Realprogymnasien	71
Realschulen	73
Total	572

Another classification:

With Latin and Greek	318
With Latin but no Greek	157
With neither Latin nor Greek	97
Total	572

We have already stated that these institutions do not correspond closely to any kind of American school, and that hence the German names will be retained in speaking of them. The ideals, administration, and discipline are *throughout* strictly those of the school, but the curriculum extends somewhat beyond that of our best high-schools. In mathematics, the work done covers approximately the same

ground as our course to the close of the freshman year in college.

Financial support.

In addition to their income from tuition fees (about $33 per annum per pupil), and from endowments (usually not large), institutions may be supported by the state, by the city in which they are located, by private organizations, or by private individuals. The state makes good deficiencies in the budget of institutions not primarily supported by it, and all institutions, even those receiving no support from the state, are obliged to conform to the same curricula and are subject to the same inspection and control by the educational authorities.

The buildings.

The school-buildings are usually located in the interior of blocks, away from the noise and bustle of the street. In Berlin they are as a rule modern and well adapted to their purpose, those of the last few decades especially being models of school architecture. There is a strange deficiency in all the buildings, even the newest, in two respects: only one blackboard, from five to eight feet in length, is to be found in each room, and the seating arrangements are not good, usually consisting of desk-benches holding from four to six pupils. When called to the board the older pupils pass before the others of the same row, while in the lower classes the boys often run

along back of the others on the benches on which they are seated. There is always a large hall for physical culture (*Turnhalle*, gymnasium in our sense of the word), fitted out with apparatus, and a yard for open-air exercise whose area (roughly estimated) is at the minimum about equal to that covered by the buildings and at the maximum is several times as great.

Each institution has two good libraries, one for the pupils and the other for the teachers. Catalogues of the first are sometimes published indicating the classes to which each book is suited. The second contains important scientific and pedagogic works, and both libraries are increased each year by purchase and gift, the titles of the new acquisitions being mentioned each year in the *Programm*. The physical and the chemical laboratories and the museums of natural history seemed to be good, but the purpose of the writer's visits did not call for careful examination of them. Mathematical models do not appear to be in much favor.

The equipment.

A large, cheerful room, called the "*Conferenzzimmer*," is always set apart for the social and official use of the faculty. Along the side walls are ranged desks and lockers for each teacher, and through the centre extend long tables amply provided with writing material, including red ink.

The faculty-room.

This room is the working centre of the institution. From here pulsates the life-current of the work, and the influence of this simple, easily provided centre gives tone, vigor, and unity to the activities of the entire teaching body. Here each teacher meets his colleagues several times daily, and exchanges a few social phrases with them or arranges some detail of their common work; here each keeps his needed books and papers, refreshes himself, rests quite free from responsibility during the ten-minute pauses and works in quiet when he has a free hour; and here finally, the Director drops in frequently for a little chat or to speak about this, that, or the other of the routine of the day. In all these ways the constant use of this room must contribute in no small degree to unity and efficiency of work.

Sessions.

The sessions are held on all six week-days and the arrangement of hours in the day is fixed by each school in accordance with local needs. The vacations vary slightly, but usually are about twelve weeks in length, divided as follows:

Easter	16 days
Whitsuntide	5 "
Summer (usually including July)	38 "
Michaelmas	9 "
Christmas	16 "
Total	84 days

In addition there are various single holidays, and in summer the session is often omitted or abridged on hot days.

The year is divided into two semesters, the Michaelmas and the Easter; if the number of pupils is large enough, there are two full sets of classes, in one of which the year begins at Michaelmas, in the other at Easter. When this is the case, a pupil who fails to be promoted falls back only six months. When there is only one set of classes the year usually begins at Easter, though not invariably.

An interval of ten minutes intervenes between the ringing of the bell which terminates one exercise and the ringing which is the signal for the beginning of the next. **The pause.** During each of these pauses the pupils are required to leave the room and take exercise in the yard; if the weather is bad they remain in the corridors. A pupil remains in charge of the books and clothing left in the classroom, and in each corridor and in the yard a teacher exercises supervision, but otherwise all are free during the pause. It is a period of relaxation and refreshment for all. Both pupils and teachers bring a supply of sandwiches to be eaten in the pauses. Noises are tolerated freely both in the corridors and on the playground; the pupils run about and shout, but I saw no concerted plays. Indeed, the American visitor

to Germany is soon struck by the fact that the children gambol about aimlessly and do not play games. As a few minutes are necessary for the pupils to return to their rooms after the bell has rung, the actual period of instruction of each "hour" is about forty-seven minutes.

VII

The Curricula

The first plan of studies uniformly followed throughout Prussia was issued in 1837. Prior to that only the rules for final examinations (*Abiturientenexamen*), first issued in 1812, and isolated ministerial orders, were binding upon the schools, which on the whole arranged their curricula independently. The class system was introduced throughout Prussia in 1820, and there have been during the present century four revisions of the course of instruction—*viz.*, in 1837, 1856, 1882, and 1892.

First curriculum.

The last revision was preceded in the autumn of 1890 by a *Conference* called by the minister of education to discuss a number of questions submitted by him. The forty-four members included representatives not only of the schools concerned, but also of the Universities, the Educational Administration, the Church (both State Church and Roman Catholic), and the Army. Among the members who are well known in America were Paulsen, Helmholtz, Virchow, and Zeller. The delibera-

The conference of 1890.

tions of the conference extended over several weeks and were opened and closed with an address by the Emperor in person. The complete stenographic report of all the deliberations has since been published[1] and will repay careful study.

Among the important problems considered were the question of obligatory Greek (the specific question at issue being further prerogatives for the real-institutions), the extent of the Latin instruction, and the overburdening of the pupils.

The first resulted in no additional prerogatives for the real-institutions; the second, in a marked diminution of the number of hours of instruction devoted to Latin, an increase in the number of hours given to German, and the abolition of the Latin theme as an end in itself (*Zielleistung*); and the third resulted in a diminution of the hours of instruction in all the schools (see curricula below), and in a sharp demand that home-work should play a very minor part (see methods, etc., below).

It is with much regret that this mention of these instructive deliberations is permitted to suffice. The interested reader will find the published report very suggestive.

Passing to the consideration of the curricula

[1] *Verhandlungen über Fragen des höheren Unterrichts*, Berlin, 1891, pp. 800.

themselves, we give first the detailed curriculum of the gymnasium as fixed by the plans of 1892. The abbreviations already mentioned (p. 34) are used for the classes, and the figures denote hours of instruction per week throughout the school year.

The curriculum of the gymnasium.

CURRICULUM OF THE GYMNASIUM.

	VI.	V.	IV.	IIIB.	IIIA.	IIB.	IIA.	IB.	IA.	Total.
Christian Religion	3	2	2	2	2	2	2	2	2	19
German	4	3	3	2	2	3	3	3	3	26
Latin	8	8	7	7	7	7	6	6	6	62
Greek	..	..	..	6	6	6	6	6	6	36
French	..	..	4	3	3	3	2	2	2	19
History and Geography	2	2	4	3	3	3	3	3	3	26
Mathematics	4	4	4	3	3	4	4	4	4	34
Natural History	2	2	2	2	..	..	..	..	..	8
Physics, Chemistry, and Mineralogy	..	..	..	..	2	2	2	2	2	10
Writing	2	2	..	..	..	..	..	..	..	4
Drawing	..	2	2	2	2	..	..	..	..	8
Total	25	25	28	30	30	30	28	28	28	252

NOTES.

1. German and Latin in VI. and V. are to be taught by the same person if possible.

2. Three hours per week of physical culture are required of all classes.

3. The portion of the table to the left of the heavy line constitutes the curriculum of the progymnasium.

4. The instruction in the Christian Religion is given according to the tenets of the State Church (Protestant) or those of the Roman Catholic Church, or both, as circumstances may demand. Jewish pupils and others who cannot conscientiously attend either the Protestant or the Roman Catholic instruction are dispensed from the requirement in the Christian Religion, on satisfying the authorities that they are receiving an equivalent amount of instruction in tenets of their own religion. If the number of Jewish pupils is sufficient, instruction in the tenets of the Jewish religion is given in the school, in the amount required by the curriculum.

The above plan indicates sufficiently clearly the way in which the hours in each subject are distributed among the various classes. In the other kinds of institutions and in the earlier curricula the distribution is, in principle, the same, so that it will suffice to give for them the totals corresponding to those contained in the last column of the above table. (The unit is one hour of instruction each week throughout one year.)

Synopsis of other curricula.

CURRICULA OF THE PRESENT CENTURY.

	Gymnasium.				Real-gymnasium.			Oberreal-schule.	
	1837	1856	1882	1892	1859	1882	1892	1882	1892
Christian Religion	18	20	19	19	20	19	19	19	19
German	22	20	21	26	29	27	28	30	34
Latin	86	86	77	62	44	54	43	..	..
Greek	42	42	40	36	..	..	..	..	..
French	12	17	21	19	34	34	31	56	47
English	..	..	..	..	20	20	18	26	25
History and Geography	24	25	28	26	30	30	28	30	28
Mathematics	32	32	34	34	47	44	42	49	47
Natural History }	16	8	10	8	18	12	12	13	12
Physics }		6	8	10*	8	12	12	14	13
Chemistry	..	..	..	..	8	6	6	9	11
Writing	..	6	4	4	7	4	4	6	6
Drawing	6	6	6	8	20	18	16	24	16
Total	258	268	268	252	285	280	259	276	258

* Includes the elements of chemistry and mineralogy.

The ministerial rescript offers the following regulations concerning the instruction in mathematics in the gymnasium:

a. **General aim of the instruction.**—Facility of calculation with numerical quantities, and their application to the usual circumstances of everyday life. Literal arithmetic to the binomial theorem for positive integral exponents and algebra to quadratics, both inclusive. Plane and solid geometry. Plane trigonometry. The idea of coördinate and some of the fundamental properties of conic sections. In all of these subjects not simply an intelligent knowledge of the theorems is to be reached, but also skill and facility in their application.

The curriculum in mathematics.

b. **Scope of instruction.**—The topics treated in the various classes are as follows:

VI. *Four hours per week.*—Review of the fundamental operations with whole numbers, both abstract and denominate. German weights, measures, and coins, with exercises in the decimal notation and the simplest calculations with decimals.

V. *Four hours per week.*—Divisibility of numbers. Common fractions. Simple exercises in proportion, to be solved by reduction to unity. German weights, measures, and coins as in VI.

IV. *Four hours per week.—Arithmetic*, two hours. Decimals, simple and compound proportions with integers and fractions (exercises from practical life). *Plane geometry*, two hours. The straight line, angles, triangles.

IIIB. *Three hours per week.—Literal arithmetic*, one hour. The fundamental operations with absolute numbers, restricted to the most necessary matter. In the exercises equations of the first degree are also to be used. *Plane geometry*, two hours. Parallelogram. First part of circle.

IIIA. *Three hours per week.—Algebra* (first half-year, one hour; second half-year, two hours). Equations of the first degree with one and several unknowns, herewith exercises in fractions. Powers with positive integral exponents. The most necessary things concerning radicals. *Plane geometry* (first half-year, two hours; second half-year, one hour). Theorems concerning quality of areas of figures. Computation of the areas of rectilinear figures. Beginning of the theory of similarity.

IIB. *Four hours per week.*—Equations, including simple quadratics with one unknown. Powers with negative and fractional exponents. Concept of logarithms. Exercises in computations with logarithms (five place). Computation of circumference and area of circle. Definition of trigonometric functions. Trigonometric computation of right and isosceles triangles. The simple bodies, with computation of lengths of edges, surfaces, and volumes.

IIA. *Four hours per week.*—The theory of powers, roots, and logarithms. Equations, in-

cluding quadratics with several unknowns. Arithmetical and geometric progressions. Conclusion of the theory of similarity, golden section, something on harmonic points and pencils. Plane trigonometry, with exercises in the computation of triangles, quadrilaterals, and regular polygons.

IB. *Four hours per week.*—Review (by means of exercises) of the algebraic work of the earlier classes. Compound interest, annuities, imaginary quantities. Completion of trigonometry (addition theorem). Solid geometry and mathematical geography on the sphere.

IA. *Four hours per week.*—Binomial theorem for positive integral exponents. Conclusion of solid geometry. The notion of co-ordinates and some fundamental properties of conic sections.

The degree of thoroughness with which these various topics are handled may be judged from the texts used, the actual work seen in the classroom, and the examinations set at the close of the course. The specimens of the examination papers which we shall give later will perhaps enable the reader to form his own opinion as to the scope of the instruction. The text-books and the class instruction will also be discussed in another connection; it may be said here that all the available information seems to indicate that on the whole the German work is at least

equal in extent and thoroughness to that of the better American schools.

c. **Methodic remarks.**—The ministerial rescript adds a few directions under the title "Methodic remarks," among which are the following:

The teaching of arithmetic is to aim at security and facility in operations with numbers. That it may be in harmony with the following algebraic instruction and prepare for it, the reviews of the fundamental operations in Sexta, as well as the treatment of fractions in Quinta and Quarta, must be based upon mathematical form, and the handling of parentheses must likewise be continually practised. In fractions the pupil is to be taught to operate with fractional parts as concrete things. The instruction in arithmetic as such stops in Quarta, but security in computation is to be maintained by continued numerical exercises in the algebraic instruction of the following classes.

Strict adherence to the work assigned to each year is an absolute requirement. As it is more difficult in mathematics than in other subjects to replace deficiencies in elementary attainments by private industry, and as experience has shown that the difficulty which this subject sometimes presents to the pupil in the upper classes is almost without exception due to deficiencies in the foundations, conscientious strictness in the promotion of pupils becomes

the more an urgent duty toward the pupils themselves.

We pass to the mathematical curriculum of the Realgymnasium and of the Oberrealschule. The scope of the instruction in the real gymnasium is as follows:

Mathematics in the "Real" schools.

Algebra, including the proof of the binomial theorem for arbitrary exponents and the solution of equations of the third degree. Plane geometry, including the theory of harmonic points and pencils, and points and axes of symmetry. Solid geometry and the fundamental propositions of descriptive geometry. Plane and spherical trigonometry. Introduction to the theory of maxima and minima. Plane analytic geometry.

In addition to these, there are required in the Oberrealschule:

The most important series of algebraic analysis. Equations of the fourth degree and the approximate numerical solution of algebraic and transcendental equations may be taken up at the option of the instructor.

In all these topics the work is to give practice in the application of the theorems, as well as to lead to a mastery of the proofs themselves.

The directions for the work in mathematics have been very sharply criticised by the mathematicians in the schools affected. It is not necessary to reproduce these criticisms here or

to make comments of our own with the one exception of calling attention to the sad deficiency in the "General Aim of the Instruction" which is set up to guide the teacher in his work. No unifying principle is offered for making the mathematical work a harmonious whole, no suggestion of treating this subject as a portion of general culture; nothing but the attainment of a specified amount of mathematical technique is fixed as the aim of the work.

German strictures.

The distribution among the various classes of the hours allotted to mathematics in the different curricula, appears in the following table:

Distribution of the hours.

Gymnasium.

	VI.	V.	IV.	IIIB.	IIIA.	IIB.	IIA.	IB.	IA.	Total.
1837.....	4	4	3	3	3	4	4	4	4	33
1856.....	4	3	3	3	3	4	4	4	4	32
1882.....	4	4	4	3	3	4	4	4	4	34
1891.....	4	4	4	3	3	4	4	4	4	34

Realgymnasium.

1859.....	5	4	6	6	6	5	5	5	5	47
1882.....	5	4	5	5	5	5	5	5	5	44
1891.....	4	4	4	5	5	5	5	5	5	42

Oberrealschule.

1882.....	5	6	6	6	6	5	5	5	5	49
1891.....	5	5	6	6	5	5	5	5	5	47

VIII

The Instruction in Mathematics

The arrangement of the classroom, the benches and the small blackboards, have already been mentioned. Wet sponges are used as erasers. Light (*always* from the left of the pupils), heat, and ventilation were as a rule adequate. The lower classes which I saw had from thirty to thirty-five pupils, while in the three upper classes the number ran from nine to twenty (the last number was exceeded in only two instances, Obersecunda with thirty pupils). The pupils are all assembled when the teacher comes and when he enters they invariably rise and remain standing until he bids them to be seated. They assume the same attitude when he leaves the room at the close of the hour, and also whenever any other member of the faculty, or a visitor like myself, enters or leaves the room. The pupils are addressed as *Du* to the close of Obertertia, thereafter as *Sie*.

Classroom customs.

The first thing which impressed me in the classwork, and that which remains finally the most prominent characteristic, was that the *teacher*

teaches. He does not "hear recitations;" he does not examine the pupils to see whether or not they have learned some assigned matter from a book; this custom seems happily quite a thing of the past here. At times he imparts new knowledge himself, especially by way of definition and introductory work, but most frequently he leads the pupils on by skilful questions themselves to discover new truths. In the development of new propositions the teacher guides the work, but the pupils suggest step by step what is to be done next.

The teacher teaches.

Home-work and the study of books are very minor features; by far the heaviest stress is laid on the class-exercise. Here, under the carefully planned instruction, under the direct influence of the personality of the teacher, the progress is to be made. Private work and the study of text-books have simply the purpose of fixing in mind or giving practice in that which is supposed already to have been learned.

Stress on the class-exercise.

The teacher is the source of the pupil's knowledge and the authority on which he builds. "What does the book say?" is a question never raised in a German school; in all my visits, I heard no books referred to, except collections of exercises.

Whether it is best to train pupils to such

marked dependence on personal, oral guidance in the acquisition of knowledge is open to question. I am here and throughout simply stating the facts as I found them without discussing their merits either when regarded alone or in comparison with other systems. An educational system is far better judged by the results it accomplishes under fair and sufficient trial than by any quantity of theorizing about it. The results reached under the Prussian system will be discussed later.

This is, however, the proper place to say that the Prussian system does produce most excellent teachers. The thorough preparation which they are required to undergo both in the scientific and the practical pedagogic lines tells distinctly in their work of instruction. There were many remarkably strong teachers among those whose classes I visited, and among them all there was only one that could be called unquestionably poor.

"The Socratic method."

If I were to describe the method of instruction taken as a whole by a single phrase, I should say it is the "Socratic method," the method of skilful questioning, of leading the class on to the desired goal by a series of questions, each usually fairly easy to answer in itself. Except in case of review-questions to refresh the memory or to recall the material needed for the day's

work, the questions have a clear didactic purpose and value, and generally give evidence of having been carefully planned. Every bit of the hour's work is vitalized by the teacher; there is not a minute when his voice is not heard and there is also not a minute when his voice only is heard.

I was especially impressed with the general custom of dividing the work into very simple steps, and of repeating each new fact established over and over until it seemed that it must be imbedded in the mind of the slowest, before going on to the proof of the next. This is very distinctly characteristic of the German classroom. The questions are very simple, often half suggesting the answer, but still leaving something for the pupil to think out and add. One director, in praising his leading teacher of mathematics most highly, said:

"He hammers away and simply *makes* the pupils follow the work; the secret is that he *works for the slowest*."

It may seem that this mode of procedure is to the disadvantage of the brighter pupils by holding them back to the pace set by the weaker portion of the class. Indeed, several German mathematicians have remarked to me upon the slowness of the progress and have recalled how irksome this had sometimes proved to them as pupils. The problem of how to care for the

weak and the average pupil without holding back the talented pupil to his detriment seems to be as far from solution in Prussia as in America. If, however, the galaxy of mathematicians who have sprung from the benches of the German gymnasia be taken into consideration, the question may well be raised whether or not the retardation of the gifted pupils is in fact to their detriment.

The answers are always given in complete sentences, and clear and distinct enunciation is insisted upon. Every lesson in mathematics is thus more or less of a lesson in German. Considerable stress is also laid upon the oral solution of easy exercises. In review, quite complicated problems are thus proved. I heard, for example, boys about thirteen years old (*Untertertianer*) prove the Pythagorean theorem with no figure whatever before their eyes. Different pupils did not use exactly the same lettering for the figure. The teacher informed me that this was not a mere feat of memory, but that the pupils could follow the proof on an imagined figure and that they enjoyed this kind of work. I saw this in various forms in several classes, and the pupils enter into it with considerable zest. Simpler new theorems are also proved orally, in some cases with the figure on the board; in others the figure was constructed and all were allowed

Oral work.

to take a good look at it and fix the image in mind, when the figure was erased before the proof was begun.

Work in concert is effected in the *Diarium* or exercise-book. Some exercise is dictated by the teacher; the pupils work it simultaneously, one reading as he works; the same pupil reads only one or two steps, so that quite a number are called on before the exercise is finished. The reply which the pupil makes seems to be the only means which the teacher has of determining in how far each pupil has worked along with the others and understood the step taken. The exercise-books are not usually inspected either during or after the class-work. Sometimes the teacher or a pupil works on the board, the others working along on paper, or looking on and dictating in response to the questions of the teacher; sometimes the teacher works on paper with the class; sometimes he has the exercise in hand, already worked out. Sometimes the result found is discussed, reformulated by members of a class until a satisfactory form is reached, and then copied as a theorem for future use. The "Diarium method" could readily be adapted to work at the board by the whole class in concert.

The exercise-book.

Whether working in the diarium or at the blackboard, the pupils are trained from the

very beginning to read aloud distinctly what they write *as they write it*—to "chalk and talk." This habit might well be cultivated in American pupils. One of the chief advantages of an oral explanation is that one sees the proof growing and taking shape, and comments can be made on each step as it is taken and its correctness and purpose satisfactorily explained before the next step is made. In the same way, figures may be constructed line by line as required. Indeed, by far the best way to master a proof which is put before one in complete form is to write out the proof step by step on paper, constructing what figures may be needed as the proof proceeds. If complete figures are prepared beforehand at the board, and the reasoning is written out in full before any oral explanation or discussion takes place, the possibilities of oral work are very imperfectly utilized. In an oral explanation the word of mouth is of chief moment; the writing is auxiliary and of the character of a record of what *has been* explained.

"Chalk and talk."

The attainment of any degree of skill in the threefold activity of performing an operation, describing it orally, and recording it in symbols, requires systematic and persistent practice throughout the entire work in mathematics. The German boys in the lowest classes were as ready in writing and reading simultaneously

the operations they had to perform as were the boys in the higher classes in their more complicated work. This is so because the teachers insist on it from the very beginning, by example as well as precept.

It might be well to encourage the pupils to practice at home explaining proofs aloud and writing them out simultaneously. In addition to its direct purpose, this practice would also contribute to fixing the spirit and methods of the proofs themselves more firmly in mind. It should therefore be confined strictly to proofs which have been fully explained in class and clearly understood, lest, otherwise, errors and erroneous conceptions should become more firmly rooted.

The following lessons may illustrate some of the characteristics of the instruction that have just been described. The first, a lesson in algebra, is taken from my notes of a class visit, while the second, a lesson in geometry, is a model for the study of beginners, set up in a German work on the teaching of mathematics.[1]

A lesson in algebra.

The algebra lesson was given in Untertertia, the first year in which algebra is taught, the minimum age of the boys on entrance into this class being eleven years.

[1] Reidt, *Anleitung zum mathematischen Unterricht an höheren Schulen*, 1886, p. 31 *et seq.*

First, the following problem from the book of exercises in algebra[1] used in this class was taken up.

To solve: $\frac{x}{2}-\frac{x}{3}+\frac{x}{4}-\frac{x}{6}+\frac{x}{8}+\frac{x}{12}=11.$

All wrote the problem, one (John, say) reading aloud as he wrote and adding:

"We seek first the common denominator."

Teacher. "How do we do that? By a rule?"

John. "No, by inspection."

Teacher. "Right. What is the common denominator?"

John. "Twenty-four."

Teacher. "Right. What do we do next, Henry?"

Henry. "We multiply both members by twenty-four."

Teacher. "What is the result, William?"

William reads as all write,

"$12x - 8x + 6x - 4x + 3x + 2x = 264$."

Teacher. "What do we do next, Karl?"

Karl. "We unite the terms in the left member."

[1] Bardey, *Methodisch geordnete Aufgabensammlung, mehr als 8,000 Aufgaben enthaltend*, 23te Auflage, 1897.

Teacher. "Give the result, Fritz."
All write as Fritz reads and writes,

$$\text{"}11x = 264.\text{"}$$

Teacher. "What do we do next, Peter?"
Peter. "We divide both sides by eleven."
Teacher. "What is the result?"
Peter reads and all write,

$$\text{"}x = 24.\text{"}$$

A problem just like this was worked similarly, and then, as this was one of the days on which home-work in mathematics is to be assigned to this class, three problems of precisely the same nature were assigned by page and number from the book of exercises for home-work, *viz.*:[1]

$$x - \frac{3x}{2} + 9 = \frac{2x}{3} + 4 + \frac{5x}{6} - \frac{6x}{5} + \frac{1}{5}.$$

$$2\tfrac{1}{3}x - 3\tfrac{1}{2}x + 5\tfrac{1}{3}x - 3\tfrac{1}{6}x + 1 = x.$$

$$2\tfrac{2}{3}x + \frac{x}{3} = 2\tfrac{1}{2} + x - 4\tfrac{1}{6}x + 5\tfrac{1}{2}.$$

This constituted the entire assignment for home-work.

[1] Bardey, *Aufgabensammlung*, p. 101, Nos. 73, 74, 75.

Next, the following problem was taken up, all writing and one reading as usual.

$$\tfrac{2}{3}(7x - 10) - \tfrac{1}{2}(50 - x) = 20.$$

Teacher. "What doesn't please us here?"

Various pupils raise hands and reply as called on by the teacher.

"The parentheses."

"The known numbers on the left."

"The fractions."

Teacher. "Which shall we remove first?"

The pupils express different opinions. The teacher points out that the most practical order must be determined in each problem—"a matter of feeling"—and then indicated that in this problem it would be easiest first to remove the fractions, then the parentheses, and then to rearrange and solve. All of this was carried through step by step as in the previous case.

Then oral work was taken up. First, the expansions of $(a + b)^2$ and $(a - b)^2$ were rehearsed both as formulæ and in words, and then a number of exercises were given, the teacher writing on the board and the pupils reading the results as called upon to do so. Very easy exercises were given at the beginning while those at the close were of the difficulty of the following:

$$(x^4 - 2y)^2;\ (5x^2 2 + x)^2;\ (a^2 + b^7)^2;\ (x - x^4 - 2y).$$

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Then the formulæ for $(a + b)^3$ and $(a - b)^3$ were deduced and repeated a number of times, and the hour came to a close.

A lesson in geometry.

The geometry lesson is supposed to be given to the class *Quarta*, in the first year's study of geometry, the minimum age for admission to this class being eleven years.

"The teacher draws a triangle ABC upon the board, and then questions the pupils somewhat as follows, the pupils being called on singly by name in as lively alternation as possible:

How many angles has a triangle?

Name an angle of the triangle ABC. A second. A third.

The teacher draws and defines an *exterior* angle, CAD.

Who can draw another exterior angle? (Done repeatedly by various pupils.)

How many exterior angles can be drawn at one vertex of the triangle?

How many exterior angles can be drawn altogether?

What are two exterior angles at the same vertex called with regard to each other?

What therefore do we know as to the magnitude of these two angles?

How many exterior angles differing in size can a triangle have at most?

Why is it customary to speak of only one exterior angle at each vertex of a triangle?

In view of this custom, how many exterior angles would a triangle be said to have?

For convenience, the letter at any vertex shall be used to denote the interior angle and the letter primed the exterior angle at that vertex. The notion of adjacent angles is supposed to have been explained earlier in the course.

What are an interior angle of a triangle and its adjacent exterior angle called with respect to each other?

What theorem holds for two such angles?

We wish now to compare the magnitude of an exterior angle with that of the two non-adjacent interior angles. For this purpose, we regard AB and AC as two non-parallel straight lines cut by a third, BA; the last produced forms the angle CAD or A′.

In what position are A′ and B with respect to each other? Likewise A′ and C?

Can therefore A′ = B, or A′ = C?

To compare the magnitudes of the angles we draw AE parallel to BC and divide A′ into the angles CAE and EAD, which angles we call x and y respectively.

Since AE is parallel to BC, there is another angle in the figure equal to y; what is it?

Why are y and B equal? Which are the parallels and which is the secant line?

The teacher marks the equal angles with the same mark.

Is there also another angle in the figure equal to x? What is it? The teacher also marks these equal angles with the same mark, different from that used with the previous pair.

Why are these two angles equal? Which two lines are now the parallels? Which the cutting line?

Since $y = B$, and $x = C$, how large is $x + y$?

To what is therefore the angle A′ equal?

What theorem have we thus found?

The proof is now repeated synthetically, first with the same figure, then with a different exterior angle of the same triangle, or also with an entirely different triangle, something as follows:

The teacher produces BC beyond C and asks:

What do we assert concerning the exterior angle at C?

What auxiliary line shall we draw to facilitate the proof?

What pairs of angles are now equal? Why? (To avoid breaking the main course of thought, the parallels and the secant line are not now called for in detail.)

What follows from the equality?

Karl, give the entire proof once more. (To hold the attention of the other pupils the teacher

interrupts, if necessary, and calls upon others to give the reasons for statements made by Karl.)

In the next hour one or more repetitions of the proof are given in the same way until, if possible, all the pupils are able to present the proof in a connected manner."

"Many a beginner in teaching will regard a large part of the questions in the above specimen as very superfluous and as uselessly squandering time, because he thinks it may be taken for granted that the pupils know the answers. He will even fear that such seemingly trivial questions will not only not attract the pupil, but actually bore him and cripple his interest in the work through default of rapid progress to new matter. But if he makes the experiment of teaching in the manner indicated, he will soon become convinced of his error. He will see that the great majority of the pupils are eager to participate in the discussion, that they compete for the privilege of answering the questions asked, and are rejoiced to know and to do something. He will also notice that the answer to these questions are by no means evident to some pupils, and he will be obliged to take special pains with these pupils. He will often enough be amazed at the colossal stupidity of some of the answers to his questions, but he will also be gratified to see

Objections considered.

how, gradually, stupidity gives place to comprehension, and the progress of the pupil, at first so slow, becomes more and more rapid, in consequence of the pupil's own mental exertions. It will not be long before the stronger pupils at least will be able to make simple proofs without the previous assistance of the teacher."

Manner of the teachers.

The manner of the teachers was usually of a military sharpness, though not unkind; in many cases it was mild, in a few *genial*, and only exceptionally unkind or irritable. The routine directions, especially, are given with the snap and precision of a military command and are met with an obedience equally military in its promptness and unanimity. This custom, together with the fact that the time spent by a teacher with the same class is measured in years, permits the development of an informal code of directions (words and even gestures), by which the time occupied in giving and executing routine directions may be considerably reduced. There was often a sharp rattling fire of questions to which the answers came with corresponding promptness and precision, but at crucial points, where it seemed necessary for the pupil to collect his thoughts, ample time was allowed. The pupils were not only not hurried, but were openly encouraged to take time to think.

"Take five minutes if you wish, *only* make no mistake."

In commenting upon the work of pupils both the warmth of the praise for the good work and the severity of the censure for the poor work were more intense than they would have been under the same circumstances in America. It is safe to say that, despite the fact that many of the teachers of mathematics in our secondary schools are women, the emotional treatment of the instruction in Prussia is decidedly more demonstrative than here.

Concerning the written exercises nothing need be noted except the neatness of the papers. I saw a number of sets, both of final examinations and of class-work, and was present a few times when the latter were returned with criticisms. All are written on uniform paper and any lack of neatness and mechanical accuracy is sharply criticised when the papers are returned. The result is that even the papers of the final examination, when perhaps hurry and flurry might palliate careless writing, are to my American eyes models of neatness. This standard can be reached only by unremitting insistence upon neatness from the very beginning and throughout the entire nine years. Indeed, I was told that the few papers which I singled out in one case as not up to the standard of the others,

Neatness of written work.

were written by pupils who had entered the Gymnasium late and thus had not had the same drill as the others.

The schedule of instruction is arranged so that any one class passes through the hands of as few different instructors in each subject as is practicable. During the nine years of the course of study the pupil has only one or two instructors in mathematics in the smaller institutions and from two to four in the larger institutions. Even in the latter, the aim is, if possible, to arrange the schedule of instruction so that the work in mathematics of the last three years shall be under one instructor, in order that the pupils may be carefully and systematically prepared for the final examination.

Homogeneity of instruction.

Whatever the number of teachers, the entire mathematical education of the boy from the elements of arithmetic to those of analytic geometry takes place in one institution under one management, guided by the close supervision of the same director and under the tuition of men of the same scientific training, who are colleagues working in close contact, with opportunities for intimate interchange of ideas. Besides the director, the senior professor of mathematics gives more or less attention to the work of the younger instructors and thus contributes to uniformity of tone and spirit.

On the desk of each class there lies a large book, very durably bound, devoted to the record of the class's work. One page is allotted to each day's work and one book serves a year. The page is provided with four columns, respectively for absentees, remarks, record of matter treated in the hour, and assignments. Horizontally, the page is ruled for each school-hour, so that at the close of the day a complete record of the day's work in the class appears on the page. The homework assigned being recorded, each instructor can see what the class is already required to prepare, and govern his own assignment accordingly. Usually, the days on which homework may be required at all are fixed for each subject, and I have at times seen this schedule posted on the wall of the classroom together with the hour-schedule for the class which is invariably there. The class-book is filled out beforehand as far as possible (dates, hours, subjects) by the *Primus* or first boy of the class. The other entries are made and signed by the teacher immediately at the close of the hour. In the column for remarks, anything not in the usual routine is entered, such as pupils excused, pupils misbehaving, reprimanded, punished, or doing very bad work.

The class-book.

A few specimens will best show the nature of these entries. They are taken from some

class-books which I was kindly allowed to look through at leisure. They are all taken from different dates during one year, and were made in one institution by various teachers in different classes.

Specimen entries.

A. was excused at 9.55. (The hour closed at ten.)

B. occupied himself with outside matters during the hour.

C. is not prepared.

D. does not know the Homer verses by heart (the second time).

D. does not know the Homer verses again (the third time).

E. was reprimanded because of disturbance during the pause.

F. is to blame for repeated lack of industry in Homer. The class was very noisy during the pause.

G. knows nothing, and besides that answers pertly.

H. is punished with one hour "*Karzer*" (imprisonment) because of persistent lying.

J., on account of repeated unbecoming conduct, receives the order to clean the sponge thoroughly daily.

K. speaks with his neighbor during the written test and is punished with one hour "*Karzer*."

The class-book is under the constant inspec-

tion of the ordinary, who thus keeps informed as to the work of the class, and the director is required to inspect all the class-books at least once each week. The pupils are also at liberty to examine it.

All home-work is regarded as supplementing the work in the class and not as an integral part of the course. Its purpose is either the cultivation of neatness and orderliness in making clean copies of class-work, the memorizing of indispensable material, the fixing of what has already been learned, or the training to independent activity. Matter that has not been thoroughly explained in the class, so that the class as a whole understands it clearly, is never assigned to be studied privately by the unaided pupils.

Home-work.

The quantity of the home-work is to be kept as small as possible. As maxima for the various classes, the following are officially suggested:

VI.	V.	IV.	IIIB.	IIIA.	IIB.	IIA.	IB.	IA.	
1	1	2	2	2½	2½	3	3	3	hours daily.

As already mentioned, the distribution of this time among the different instructors is arranged beforehand by the director or by the instructors interested in conference, fixing the days on which each instructor may assign any home-work and the maximum of time which he is then at liberty to require. The permanent record of

every assignment in the class-book enables colleagues and superiors to keep track of each instructor and see that he does not exceed his allotment.

Specimen allotment.

I learned the assignments in a few cases, and it seems, roughly speaking, that the amount of home-work in mathematics required per week in Prussia is considerably less than twice the amount required per day in the United States. As specimen of the allotment of time for private work, the following table may serve, giving the hours per week, as fixed for each subject and class in the Kgl. Realgymnasium, Berlin.

	VI.	V.	IV.	IIIB.	IIIA.	IIB.	IIA.	IB.	IA.
Christian Religion	1½	1½	1½	1½	1½	1½	1½	1½	1½
German	1	1	1	1	1	1½	1½	2	2
Latin	3	3	4	2½	2½	2½	2	2	2
French	.....	.....	1	1½	2	2	2	2	2
English	.....	.....	.....	1	1½	1½	2	2	2
History	0	0	½	½	½	1	1½	1½	2
Geography	½	½	½	½	½	½	.....	.....	....
Mathematics	1	1	1½	2	2	2	2½	2½	2½
Natural History	½	½	½	½	½	½	.....	.....	....
Physics	.....	.....	.....	.....	.....	½	1	1½	1½
Chemistry	.....	.....	.....	.....	.....	.....	½	1	1
Total per week	7½	7½	10½	11½	12½	13½	14½	16	16½
Total per day	1½	1½	1½	1 5/12	2½	2½	2 11/12	2½	2½
Maximum per day suggested by Ministry	1	1	2	2	2½	2½	3	3	3

In noticing the allotment to mathematics, it should not be forgotten that the scope of the work and the number of hours of instruction

in mathematics in the Realgymnasium are considerably in excess of what is done in the Gymnasium.

The weekly amount is divided up into one, two, or three daily portions for each subject, and the days on which these allotments may be used are specified in such a manner that the aggregate daily assignment for private work is not far from the average on any one day. There are no "study hours" for pupils at school. While there, their time is occupied entirely with class-work.

Ministerial rescripts of December 24, 1833, and August 16, 1860, require that some text-book be used in mathematics.[1] The decision as to which book or books

Text-books.

[1] The following is a free translation of these rescripts.

(1833) "The Ministry has had occasion to note that in many Gymnasia the instruction in mathematics is conducted without a definite text in the hands of the pupils. In mathematics, if anywhere, a brief text, suited to the needs of each class, is indispensable. . . . In order to counteract these and other evils which have hitherto been more or less marked in the mathematical instruction in the Gymnasia by reason of the lack of a definite text-book, the Ministry wishes herewith to fix that from Easter of next year a definite text-book is to be used in the various classes of all Gymnasia, that this text-book is to be in the hands of the pupils, and that no further attention is to be paid to any objections to this regulation which the teachers may raise.

(1860) "The large number of text-books in mathematics and physics which are in use is an evil of considerable import. It is therefore very desirable that the use of those which have not proved

are to be used is made by the local authorities, but their choice is limited to such works as have been approved as sufficient for the purpose by the Ministry or the Provincial Board. Many books are used in one institution only. The presumption might be that in these cases the authors are teachers in the institutions using the book, but this is by no means always the case. To counteract the tendency to multiply text-books the authorities are becoming more slow in placing new books on the approved list. It is now required that institutions with which the author is not connected shall have expressed their desire and purpose to use the book as text before it is placed on the approved list. Still, with all this discouragement, the German teachers are writing new text-books in mathematics by the score every year.

The text-book adopted is in many cases regarded as named rather to comply formally with the regulation than for the purpose of actually using the book. This statement is

strong books be still further discontinued and that they be replaced by more suitable works; nevertheless, the educational administration will refrain now, as heretofore, from any direct constraint in this matter. The books adopted are often used too little, still, in attempting to enforce more use of the books, the danger would be incurred of hampering the more important effectiveness of the free individuality of the teacher."

based both on information which I obtained from German teachers and on my own observation. In all my visits I saw no books used in the classes (except collections of exercises and tables of logarithms), nor did I hear any allusion to the text in the work of the hour or in the assignment of home-work. One very excellent professor informed me, upon inquiry, that "Mehler" was indeed officially in use, but that it was sometimes not alluded to in the class-work for months at a time.

Whatever may be the extent of the actual use of the text-book, the German teachers are unanimously agreed on one point— the study of any particular topic in the text-book must *always* follow the development of that subject in the class. The chief functions of the text-book are considered to be:

Purpose of the text.

First, to avoid the loss of time of instruction involved in the pupil's copying the teacher's explanations into a book as they are given.

Second, to give those pupils who have not thoroughly understood the class presentation (and whose notes would probably also be faulty), a faultless presentation to which they refer and review the class-work, and clear up any points which had remained obscure.

The books in use are accordingly written in a more or less brief form, as a skeleton, rather than as a complete body of instruction. Those

books which do give detailed treatment usually profess to do so out of consideration of the needs of such as may wish to use the book for private study, unaided by a teacher. Of course, opinions vary widely among the teachers as to the degree of detail with which a book intended for class use should be written. Those actually in use differ much in this respect, and many teachers publish, for the use of their own institutions primarily, supplementary and fuller treatments of particular chapters, topics, or class allotments than the concise text gives.

IX

The Examinations

One of the results of the Conference of 1890, as embodied in the new curricula of 1892, is an attempt to diminish somewhat the tasks of the pupils. Among other things, the examinations have been made lighter. **The examinations.**

Annual oral examinations for promotion are held at the discretion of the Director and in his presence. So far as I could learn, these examinations are held on one or two subjects only in each class, and class-work alone determines the verdict in other subjects.

The first formal examination comes at the end of *Untersecunda* and is known as the *Abschlussprüfung*. It covers the work of *Untersecunda*, and is conducted by a commission consisting of a Royal Commissioner, the Director, and the teachers giving instruction in *Untersecunda*. The questions are prepared by the teachers of the respective subjects and approved by the Director. The written examination in mathematics occupies four hours. In other respects, this examination is conducted like the final ex-

amination, which will be described next. The certificate that the "*Abschlussprüfung*" has been passed entitles the holder to a diminution of one year in the length of time of his military service (*i.e.*, he serves one year instead of two).

The final examination.

The final examination, known as "*Reifeprüfung*," [1] or also as "*Abiturientenexamen*," is conducted by a Royal Commission, consisting of a Royal Commissioner (who is appointed by the Provincial School-board, and is usually that one of its members who has the special supervision of the school in question), the Director, and the teachers giving instruction in *Oberprima*. Three months in advance the pupils give written notice to the Director of their desire to take the examination. The list of those applying is discussed in a conference between the Director and the teachers in *Oberprima*. The predicate to be assigned to each for his class-work is fixed (those in use are: Very Good; Good; Satisfactory; Unsatisfactory). By unanimous vote, those whose class-work has been unsatisfactory in all subjects may be excluded from the examination.

The examination is both written and oral.

[1] In describing this examination, we give simply a sketch of its characteristic features, without mentioning the exceptions and alternatives which may arise and which are duly provided for in the regulations.

The subjects of the written examination and the time allowed in addition to the time needed to dictate the papers set are as follows:

The written examination.

German Theme, five hours; Mathematics, five hours; Translation from German into Latin, two hours; Translation from Greek and French into German, each three hours.

Lexicons may be used, and in mathematics a table of logarithms.

The mathematical paper contains four exercises, one each from Plane Geometry, Solid Geometry, Algebra, and Trigonometry. The papers set are afterward published in the *Programm*, and specimens will be given when the latter is described.

In each of the subjects for examination—Latin, Greek, French, German, and Mathematics—three examination papers are prepared by the teacher of that subject in *Oberprima*, and these papers, if approved by the Director, are sent to the *Schulkollegium* along with a list of the candidates and a sketch of their school-course. From each set of three the commissioner selects one to be used for the examination and returns it in a sealed envelope to be opened at the time of the examination. He may reject all and prepare the paper to be used himself, if he sees fit.

The papers are to make no requirements

whatever which exceed in kind or difficulty the work of Prima, yet they must differ sufficiently from the class-work to demand independent thinking; in mathematics, in particular, the problems must all be so-called "originals."

Independent work required.

If the papers written by the pupils are such that the Royal Commissioner doubts whether the pupils have really done independent work in the examination, he has the power to require a new examination, the paper being set by himself. The writing of the papers is supervised by the teachers of the class by turns.

The papers written by the pupils are first criticised by the teacher of the subject in question; all errors, even though very slight, are carefully corrected in red ink, and at the close the paper as a whole is characterized in a few words, and a predicate assigned by the teacher; the predicate for the class-work is also mentioned.

A few specimens will illustrate the characterizations of the mathematical papers; these general remarks are in addition to a specific predicate for each problem:

"The paper of A. is correct throughout and skilfully written, and also quite extensive. *Very good.* His class-work in papers of similar kind was *good.*

"B. has indeed made two gross errors, still

his paper may be called *satisfactory* in view of the ideas developed in it. His class-work in work of the same sort was *satisfactory*.

"C.'s paper is not free from obscurities and contains some mistakes, it is true, but it can still unhesitatingly be called *satisfactory*. His class-work has hitherto *not been satisfactory*.

"D.'s paper contains an oversight in the first problem, otherwise it is highly to be praised. *Good*. Class-work, *good*."

Next the papers are circulated for inspection among all the teachers of the Commission, who thereafter hold a conference with the Director and decide which pupils are to be recommended for exemption from oral examination. The corrected papers are then sent to the Royal Commissioner, who may alter the predicates.

The oral examination must be held within the last six weeks of the school year at a date fixed by the Royal Commissioner who presides therein. He determines the sequence of subjects for this examination and the amount of time to be given to each. All the teachers of the institution are required to attend the examination, from which are debarred those candidates whose written papers have all been "unsatisfactory," and excused those whose written papers and class-work in all the required subjects have all been at least "satis-

The oral examination.

factory." Partial exemption from the oral examination may also be granted, and accordingly only those pupils appear in the examination whose work has been unsatisfactory in part, but not entirely so. At most ten may be examined in one day.

The questions in each subject are asked by the teacher of that subject in Oberprima and by the Royal Commissioner. In the course of the examination, the predicates to be given to each pupil in each subject are fixed by the Commission, upon motion of the teacher of the subject in question, and, in conclusion, the Commission holds a consultation under the presidency of the Royal Commissioner in which the final outcome of the whole examination is determined. The pupil passes normally if none of his predicates in the various subjects (formed by combining the predicates for the class-work and the examination) are "unsatisfactory." With certain restrictions, the predicate "unsatisfactory" in one subject may be counterbalanced by the predicate at least "good" in another. This consultation is not a mere mechanical collation of predicates, but when necessary the merits of individual cases are deliberated upon as such. All the proceedings in connection with the examinations are fully recorded. The pupils who have passed receive a certificate to that effect, containing all the

predicates and signed by each member of the Commission.

The diploma of a gymnasium confers various privileges upon its holder, such as admission to University study, to technical schools, to various examinations for admission to certain military schools and the higher branches of the Government's civil service, and is also accepted in lieu of some of the earlier examinations for officers in the army or the navy. Promotion to each of the four upper classes opens to the pupil the door into some occupations which had been closed to him before. Many of the privileges attained in the gymnasium may also be attained in the Real-gymnasium and in the Oberrealschule, though some of the most important may be attained in the gymnasium only.

Privileges of the graduate.

X

The Programm

The Director of each institution publishes annually what is called the *Programm*, some-

The scientific paper.

what analogous to our annual catalogue. It is usually in quarto form, substantially, plainly, and inexpensively gotten up. The contents fall into two main parts: A scientific paper (*Wissenschaftliche Abhandlung*) and the school-report. The scientific paper may treat an advanced topic, beyond the curriculum of the school, topics from the school-curriculum, or questions of pedagogic method. It is not obligatory upon the incomplete institutions to publish this paper. In 1896 there appeared 670 such papers in all Germany, the number of institutions being 993.

The second part usually contains:

I. **The curriculum,** in a table like that which we have used above.

The school-report.

II. **The allotment** of the hours of instruction to the various teachers, in a table which shows very clearly and readily the work of each teacher, and of each class.

III. **The Class-work.**—Abstracts of the work of each class in each subject, with specification of number of hours per week and name of teacher. The abstracts in other subjects are given with about the same degree of detail as those in mathematics, of which we give the following specimens for the four upper classes of two institutions taken at random.

Humboldt Gymnasium, Berlin, 1896.

IA. Mathematics, four hours. Voss. The notion of co-ordinates; something about conic sections; maxima and minima; series; computation of the known functions. A paper to be prepared at home each month.

IB. Mathematics, four hours. Voss. Compound interest; extension of trigonometry; imaginaries; solid geometry; mathematical geography. Each month a home paper.

IIA. Four hours. Schnodt. Trigonometry; completion of the theory of similarity; construction of algebraic expressions; detailed review of powers, roots, and logarithms; equations (several unknowns) which can be reduced to quadratics; arithmetical and geometric series; reviews in trigonometry. Home paper every four weeks.

IIB. Four hours. Schnodt. Extension of the theorems on similarity; trigonometric

functions in the right triangle; fractional and negative powers; logarithms; computation of the circumference and area of the circle; formulæ for the surface and the volume of the simplest bodies; equations of the second degree. Written papers every two weeks.

Friedrichs Gymnasium, Berlin, 1896.

IA. Four hours. Fischer, I. The number *e* and the circle of numbers connected with it; supplement to solid geometry in connection with spherical trigonometry; the notion of coordinates and the fundamental properties of the conic sections.

IB. Four hours. Fischer, I. Imaginaries: combinations in application to arithmetical series of higher orders and to the binomial theorem; completion of trigonometry; solid geometry.

IIA. Four hours. Fischer, I. Powers, roots, logarithms; arithmetical and geometric series of the first order with compound interest and annuities; conclusion of plane geometry; computation of the circle; plane trigonometry.

IIB. Four hours. Schulze. Equations, including simple quadratics with one unknown; definition of powers with negative and fractional exponents; notion of logarithms; practice in the use of logarithms; computation of

the circumference and the area of the circle; the trigonometric functions; computation of right and oblique triangles; the simple bodies, with the computation of their edges, surfaces, and volumes.

Besides these are given the subjects assigned for themes in the various classes and the questions set in the written examination for graduation. These questions are scrutinized by the teachers in other institutions, and in mathematics at least form a fertile source of fresh material in the line of exercises for class use.

We give now a few specimens of the final examination-papers in mathematics in institutions of the various types.

Friedrichs Gymnasium, Berlin, Michaelmas, 1895.

1. $x^5 = 1$. The five roots are to be determined both algebraically and trigonometrically.

2. A loan of 500,000 marks is to be repaid by annual payments of 70,000 marks. How many years will be required and how much will be the last payment? (Rate of interest not specified.)

Examination-papers in mathematics.

3. In a triangle, there are given one side, $c = 748$m., the difference of the adjacent angles, $d = 11° 29.4'$, and the ratio of their sines as 4/3. The triangle is to be solved.

4. A segment of a material hollow sphere of

radius r = 533cm. has the weight 1.5 grm. per qcm. of its surface. If now this basin will barely swim in water (sinking to the rim), what is the depth of the basin and how large is the radius of the circular rim?

Humboldt Gymnasium, Berlin, Michaelmas, 1895.

1. Given a circle and an ellipse of equal area with major axis a. A rectangle is to be inscribed in the ellipse equal in area to the square inscribed in the circle.

2. Given a circle and two tangents to it. They intercept upon a third tangent the length a. To find the angle between the third tangent and one of the others. To be worked first in theory and then computed for the following numerical values: r = 6.082, a = 35.025, and the angle between the two given tangents = 67° 22.5′.

3. A body of weight q falling from height h with initial velocity c penetrates s meters into the ground. How large is the resistance of the ground? To be computed also numerically for h = 347m., c = 7m., s = 0.65m., and q = 1kg.

4. About a given spherical segment a cone of minimum volume is to be described.

Luisenstädtisches Realgymnasium, Berlin, Easter, 1895.

1. What conic section is represented by the equation (rectangular co-ordinates),

$$4x^2 - 4xy + y^2 - 6x\sqrt{\tfrac{1}{6}} - 8y\sqrt{\tfrac{1}{6}} + 2 = 0?$$

2. Solve the system

$$xu = xz = 10;\ x + y - u - z = 4;\ x^2 + u^2 + y^2 + z^2 = 130.$$

3. In the geographical latitude of Berlin, what time lapses on the longest day until the middle point of the sun is 6° below the horizon?

4. Given the radius r and the altitude h of a right cone, to determine the cylinder of maximum volume which can be inscribed in it.

Friederich Werder'sche Oberrealschule, Berlin, Easter, 1895.

1. From a triangle with sides a, b, c, a rectangle is to be cut such that the cylinder formed by bringing a pair of opposite edges together by bending a rectangle shall be as large as possible.

2. The sinking sun shines into a room through a circular opening in the front wall, and throws an elliptic spot of light on the side wall. From what point in the rear wall does this patch of

light appear to be circular? Construct this point, if the floor-plan of the room, the position of the circular opening, and the direction of the sun's rays are given.

3. By what equation is the radius of a sphere determined on which a triangle formed by three arcs of great circles has sides a, 2a, 2a, and the angle A between the equal sides?

4. Investigate the curve

$$4x^2 - 6xy + \tfrac{1}{2}\tfrac{1}{2}y^2 + 36x - 52y + 101 = 0,$$

and compute the area enclosed by it.

Eighth Realschule, Berlin, Easter, 1896 (*Abschlussprüfung—i.e.*, at end of *Untersecunda*).

1. A wooden sphere sinks in water to one-half its height and in alcohol to 7/12 its height. What is the specific gravity of alcohol?

2. On the side AB of a given quadrilateral ABCD a rectangle is to be constructed equal to the sum of the three squares on the other three sides.

3. In an arithmetical series, the sum of the second and the eighth term is 22, and the product of the third and the sixth term is 91. What are the first term and the difference of the series?

IV. **List of Text-books Used.**—Sometimes these are given in connection with the class-work

(*Pensen*); sometimes no information on this point is given. The list is often in the form of a table making clear to the eye at once what books are used in each subject, in each class, in what classes each book is used, and the price of the book. The use of text-books in mathematics has already been discussed.

V. **Orders of the superior boards,** so far as they are of general interest.

As specimens we give the following, taken from the *Programm* of the Luisen Gymnasium, Berlin, 1896. When not otherwise specified, the orders are from the Royal Provincial Schulkollegium.

1895.	
March 18.	Physician's certificate required with application for excuse from physical training.
" 21.	The 80th birthday of Prince Bismarck is to be a holiday.
April 3.	Arrangement of vacation courses in natural sciences and archæology.
" "	Dr. Fritz Bösch assigned to the institution as *cand. prob.*
May 2.	Days on which the institution has to flag.
" 20.	Approval of the rules of the boating division.
June 10.	Tickets are received for five pupils and one teacher for the Luther Celebration in the New Market.
" 20.	The work of Lindner on the war of 1870–71 is recommended.

August 2. Danger of firearms in the hands of pupils.

" 13. Approval of the leave of absence of Professor Dr. Weber.

" 27. The institution is to participate in the Sedan celebration in the Lustgarten.

" 28. Freiherr v. Mirbach sends from the Civil Cabinet of Her Majesty the Empress an invitation to the consecration of the Emperor William Memorial Church.

" 30. To illuminate on September 2d (Sedan day).

October 17. Permission to the institution to participate in the consecration ceremonies (October 21st) of the Emperor Frederick Memorial Church.

November 1. The vacations for 1896 are fixed.

" 2. The introduction of the new French schoolbook by Ploetz Cares is allowed.

" 8. Permission to add a seventh hour of Latin in Prima and in lieu thereof change one hour of physical culture into open-air games.

" 19. Notification that Professor Ewald, Director of the Art School, will inspect the instruction in drawing.

" 26. The Stenographic Association is allowed the use of a room.

December 10. Programme for the celebration of the 25th anniversary of the German Empire, on January 18, 1896. Addresses, songs, declamations.

" 17. Professor Dr. Gemss and Professor Dr. Weber receive the rank of Councillors of the Fourth Class.

" 19. The Minister presents the work of Breysig: *Brandenburgsche Finanzen.*

1896.

January 17. From the administration of the privy purse (*Schatullenverwaltung*) of His Majesty the Emperor the institution receives as present the picture "*Nations of Europe*," by His Majesty the Emperor, with autographic signature.

" " Three copies of Lindner's book, the war of 1870–71, are sent for presentation to good pupils; also three copies of the address of General von Mischke at the unveiling of the Emperor Frederick monument near Wörth.

" " Leave of absence granted to Professor Dr. Weber.

" " To illuminate on January 18th.

" 28. The work of Röchling and Knötel, *Der alte Fritz*, is commended.

February 6. From Easter on, the Jewish religious instruction is to be combined with that of another institution.

" 27. Wall-charts recommended.

The above are about half of those published in this *Programm* as of general interest, and illustrate well how thorough and detailed is the supervision by the *Schulkollegium*.

VI. **Chronicles of the Institution.**—This is a concise sketch of the life of the institution. Festal days celebrated are briefly described; occasionally an address by the Director is published in full; excursions made are enumerated; omissions of instruction for any reason specified in detail; changes in the Faculty are recorded;

if new members are appointed a full account of their previous career is given; death or serious cases of illness among the Faculty or pupils, as well as any distinctions or honors that may have come to any of the Faculty, are suitably noticed; visits of inspection by the Provincial School Councillors (members of the *Schulkollegium*) are described in detail, with words of thanks for the advice and encouragement received; in short, any deviation from the ordinary routine of the school is here made the subject of concise record, so that on one or two pages a clear picture of the school-life is painted.

VII. **Statistics.**—1. Summary of the attendance, by religion and residence.

2. Table of the attendance in the various classes, gain or loss during the year, and the average ages.

3. List of the graduates, giving for each, name, place, and date of birth, religion, years in the institution, years in Prima, occupation of father, and prospective occupation of the graduate.

VIII. **Additions to libraries, laboratories, and museums** during the year. These are enumerated in detail, and all gifts, however slight, are mentioned in connection with the name of the giver and words of thanks.

IX. **Stipends and funds** for the support of pupils. A report of receipts and expenditures.

X. **Notices to Pupils and Parents.**—Information regarding the rules and administration of the school. Under this head almost all of the *Programms* in Prussia in 1896 brought in full a long, cautionary letter from the Minister of Education concerning the danger to pupils in handling firearms which was called forth by the accidental shooting of two pupils in this manner somewhere in Prussia.

The information contained in the *Programm* is almost invariably grouped under the above heads, but their order is sometimes varied.

XI

The Reformschule

In what has preceded, the work of the Prussian higher-school system has been described in so far as is necessary in order to understand the character of the work in mathematics done in these schools, but before closing this description a few words may be added concerning two topics of some interest in this connection, the *Reformschule* and the higher education of women.

Character and purpose.

In several institutions known as *Reformschulen* the experiment has recently been inaugurated of building curricula, intended to be equivalent to the three now current, upon a common substructure up to the close of *Untertertia*, and of not separating the gymnasial and the realgymnasial course before *Untersecunda*. These institutions aim primarily to defer the decision between Greek or no Greek, Latin or no Latin, to a later period than the present curricula permit. As matters stand at present, the parents of the boy must decide which of the three classes of institutions

he is to enter when he is only nine years of age, and when once the boy has made any progress in the work of any one class of institution he can transfer to another class only with considerable disadvantage. In the *Reformschule*, the choice between Latin and no Latin need not be made until the boy is at least twelve years old, and the choice between Greek or no Greek is then deferred two years further. To accomplish this and still give an equivalent amount of Latin and of Greek it is necessary to give eight hours a week each to Latin and Greek during the last four years of the gymnasial course, and ten hours a week to Latin during the two years preceding these. The total amount of time given to Latin and Greek under this plan is not so great as that given under the present curricula, but it is believed by the advocates of the plan that the work done under the two plans will be equivalent, because the pupil should be able to accomplish more per hour in the latter part of his course than in the earlier years. To counterbalance the Latin and Greek thrown forward, French is thrown back into the earlier years, and more hours are allotted to it than under the present plans. In how far these and other expectations of the friends of the new movement will be realized remains to be seen. The ministry gives these institutions free scope for the trial

of the experiment, and the outcome is awaited with much interest.

The first institution to begin the experiment was at Frankfurt a/M., where the work was begun in 1892; since then one institution each in Hanover, Breslau, and Berlin have in turn taken up the plan. As these institutions had previously been working on the usual curricula, the transformation of the institution into a *Reformschule* must be made gradually with the progress of the classes, those which had begun work under the standard curriculum having to be carried through according to the same. The process of transformation consequently requires nine years for its completion, and the first *Reformschule*, that at Frankfurt a/M., will graduate its first class as a *Reformschule* in 1901, and the other institutions several years later still. As no institution is as yet working under the new curriculum solely, it is too early to speak definitely of the outcome, and we close with the detailed curriculum of the *Leibniz Reformschule* in Hanover, and a comparative summary table of the curricula at Hanover, Frankfurt, and the current plans of 1892.

CURRICULUM OF THE REFORMSCHULE AT HANOVER.

	Substructure.			Middle Structure.	
	VI.	V.	IV.	IIIB.	IIIA.
Christian Religion	3	2	2	2	2
German	5	4	4	3	3
Latin	..	..	..	10	10
French	6	6	6	3	3
English	..	..	..	..	..
Greek	..	..	..	..	..
History and Geography	2	2	5	4	4
Mathematics	5	5	5	4	4
Natural History	2	2	2	2	2
Physics	..	..	..	..	..
Chemistry and Mineralogy	..	..	..	..	..
Writing	2	2	2	..	..
Drawing	..	2	2	2	2
Total	25	25	28	30	30

	Gymnasial Superstructure.				G. Total.	Realgymnasial Superstructure.				R. G. Total.
	IIB.	IIA.	IB.	IB.		IIB.	IIA.	IB.	IA.	
Christian Religion	2	2	2	2	19	2	2	2	2	19
German	3	3	3	3	31	3	3	3	3	31
Latin	8	8	8	8	52	5	5	5	5	40
French	2	2	2	2	32	3	3	3	3	36
English	..	..	..	..	..	6	4	4	4	18
Greek	8	8	8	8	32	..	..	..	..	..
History and Geography	2	3	3	3	28	3	3	3	3	29
Mathematics	4	3	4	4	38	5	5	5	5	43
Natural History	..	..	..	..	10	..	..	..	..	10
Physics	2	2	2	2	8	3	3	3	3	12
Chemistry and Mineralogy	..	..	..	..	..	..	2	2	2	6
Writing	..	..	..	..	6	..	..	..	..	6
Drawing	..	..	..	..	8	2	2	2	2	16
Total	31	31	32	32	264	32	32	32	32	266

COMPARATIVE SUMMARY OF CURRICULA.

	Gymnasium.			Realgymnasium.		
	Hanover.	Frankfurt.	1892.	Hanover.	Frankfurt.	1892.
Christian Religion	19	19	19	19	19	19
German	31	31	26	31	31	28
Latin	52	52	62	40	40	43
French	30	30	19	36	38	31
English	..	..	..	18	18	18
Greek	32	32	36	..	..	..
History and Geography	28	24	26	29	27	28
Mathematics	38	37	34	43	42	42
Natural History	10	10	8	10	10	12
Physics	8	8	10	12	9	12
Chemistry and Mineralogy	..	..	..	6	6	6
Writing	6	6	4	6	6	4
Drawing	8	8	8	16	16	16
Total	264	257	252	266	262	259

XII

The Higher Education of Women

But little has as yet been done toward providing for women similar educational facilities to those for men which we have just been describing. No full-fledged gymnasia for women exist, and the proposal to found one in Breslau was vetoed by the Minister during my stay in Berlin. In the latter city an arrangement has, however, been effected whereby the "Courses of the Gymnasium" (*Gymnasialkurse*) may be taken by women under the instruction of various strong professors from some of the Berlin gymnasia.

No gymnasia for women.

What is done.

The courses are given in the afternoon when the instructors are free from their regular duties, and opportunity is given to those women who have completed courses equivalent to the curriculum of a Gymnasium to pass the regular final examination before some one of the Royal Commissions conducting the examinations of the Berlin institutions. The women who pass

the examination successfully are admitted to University lectures under certain conditions.

This work for women is under the direction of Miss Helene Lange, whose strong plea for the privilege of higher education for women is known to American readers in a translation published under the title "Higher Education of Women in Europe."[1]

The quality of the work.

Through the courtesy of Miss Lange I was permitted to attend several classes in mathematics, and found the young women doing excellent work. As might be expected, they were distinctly more mature than the boys doing the same grade of work, and were decidedly more earnest and serious in their work; the social conditions in Germany are still such that a woman who seeks a higher education is regarded as a pronounced "bluestocking," and this state of affairs deters all but women of fixed purpose and strong character from taking up these courses. With all the social traditions and prejudices discountenancing this form of education for women and just as strongly urging it upon men, and even making it absolutely prerequisite to nearly all of the most desired careers, it is not surprising that, while boys of all degrees of talent flock to the gymnasia, only women of marked strength

[1] Lange, *Higher Education of Women in Europe*, D. Appleton & Co., 1890.

are brave enough to carry through the corresponding work.

Miss Lange has pointed out in the book mentioned above that Germany is behind the other European nations (not to mention America) in the paucity of the facilities for the acquisition of a higher education which it offers to women, but progress is being made, and year by year the privileges of women in this regard are being increased steadily, even though slowly.

Progress being made.

XIII

Comparison between German and American Work

We are now in a position to consider in detail the facts upon which was based the assertion made at the beginning of this report—*viz.*, that *in the corresponding nine years we Americans accomplish no more in mathematics than do the Prussians, and that we use up seven-fourths as large a fraction of the time of instruction in doing it as do the Prussians.*

Basis of comparison.

As the American basis of comparison the Chicago schools (Grades, High-Schools, and the University of Chicago) were taken, in the belief that they are on the whole as typical of the better class of American schools as any concerning which facts could readily be obtained, and that their work is at least up to the average of that done in institutions of similar character throughout the country.

The nine years in the Chicago system which correspond to the nine years of the gymnasial course are readily seen.

At the age of nine the boy enters the gymnasium, and here at the age of nine he enters the fourth grade and has had something of fractions, decimals, and denominate numbers in addition to what the German boy has had.

Sketch of American course.

He continues arithmetic during the next five years to the close of the eighth grade, but a part of the time allotted to mathematics in the latter grade is devoted to an introduction to algebra.

In the first three years of the High-School, algebra through quadratics, plane and solid geometry are taken up; during the fourth year of the High-School no time is given to mathematics, and consequently the first year in college constitutes the ninth year of mathematical work; in this year plane trigonometry and college algebra are given twelve weeks each.

The nine years of American school-work which are thus brought into comparison with the nine years of the German gymnasium are the five years next preceding the High-School, the first three years of the High-School, and the Freshman year in college. In the last year the American has one year's advantage over the German as to age and mental development, but he has also the disadvantage of taking up his work in mathematics after having suspended it for a year.

If the German curriculum as given above be compared with the ground usually covered in the nine corresponding years in America, it will appear that the topics taken up are in substantial agreement, with the exception of the conic sections, something of which is contained in the German curriculum but not in the work of the American years in question.

Work done compared.

The text-books used and the exercises in the classroom both indicate that the German treatment of the topics is, on the whole, at least equivalent in scope and thoroughness to that in America. The test which the reader can most readily apply in this respect is to scrutinize the examination-papers which the German boys pass at the close of their work. The specimens given above may be regarded as sufficiently characteristic.

Of course the German pupils have been *prepared* to pass such papers, even though the questions are all supposed to call for independent thinking at the time of the examination, but perhaps the chief object of final examinations of this character is to test and exhibit the extent and thoroughness of the preparation of the pupils. There is little question that the German pupils would be able to pass the corresponding examinations given here.

The following table taken in connection with the preceding description of the Prussian gymnasium, will substantiate the assertion which was made at the outset. It should be noted first that the comparison is made between fractions of the total time of instruction and not between number of hours of instruction, for if four hours' instruction out of eighteen weekly be given to mathematics, more work should be accomplished per week than if four hours out of thirty are given. The smaller number of hours per week instruction implies more time for private or home-work, and mathematics is sure to secure its full share of this time. In fact, it is very possible that the average American pupil gives to mathematics more than its proportionate fraction of his private work. In Germany, the allotment of time to the various subjects, the record of assignments for home-work, and the methods and traditions of instruction all combine to make it improbable that more than the allotted portion of home-work accrues to mathematics.

Time ratios compared.

The table will be understood without further explanation. In the statement of the fraction of the total time of instruction given to mathematics the denominator is the total amount of instruction per week, and the numerator is the amount given to mathematics. The unit used is in some cases the minute, in others the hour,

TABLE OF TIME OF INSTRUCTION GIVEN TO MATHEMATICS.

Prussian Gymnasium.	VI.	V.	IV.	IIIB.	IIIA.	IIB.	IIA.	IB.	IA.	Total time for mathematics in ⅘ per year.	Average per year.
Fraction of total time of instruction given to mathematics....	4/25	4/25	4/28	3/30	3/30	4/30	4/28	4/28	4/28		
Ditto in per cent.....	16	16	14.3	10	10	13.3	14.3	14.3	14.3	122.5	13.6
American schools as specified.	IV. Grade.	V. Grade.	VI. Grade.	VII. Grade.	VIII. Grade.	First year of High-School.	Second year of High-School.	Third year of High-School.	First year in University of Chicago.		
Fraction of total time of instruction given to mathematics ...	* 300/1225	300/1225	300/1225	300/1225	300/1225	4/18	4/18	4/18	2/9		
Ditto in per cent. ...	24.5	24.5	24.5	24.5	24.5	22.2	22.2	22.2	22.2	211.3	23.5
Ratio American to Prussian	1.53	1.53	1.71	2.45	2.45	1.67	1.55	1.55	1.55	1.72	1.73
Harvard School and Univ. of Chicago.	IV. Grade.	V. Grade.	VI. Grade.	VII. Grade.	First year, Coll. Prep.	Second year, Coll. Prep.	Third year, Coll. Prep.	Fourth year, Coll. Prep.	First year, U. of C.		
Per cent. of total time of instruction given to mathematics....	13.3	13.3	12.5	13.9	22.7	22.7	22.7	21.7	22.2		
Ratio Harv'd School, etc., to Prussian ..	0.83	0.83	0.87	1.39	2.27	1.71	1.59	1.52	1.55	1.35	1.35

* In the grades IV. to VIII. each teacher has two classes, and the total time of instruction (1225 minutes per week) is ordinarily divided equally between them; each pupil thus receives 612½ minutes per week of instruction, of which 150 are given to mathematics.

and in the University the *Course* (of sixty hours). It will be apparent from the fraction which unit has been used.

For purposes of comparison the table contains also the data for the Harvard School of Chicago, which gives under one administration the first eight of the nine years in question.

XIV

Conclusion

Having now set forth the facts upon whose warrant it was asserted at the outset that a great disparity exists between the portions of time consumed in Germany and in America in doing work in mathematics which is in the main equivalent, it remains only to consider the influences which seem to contribute to this difference. The discussion of some of these merely emphasizes the fact that these influences seem to be well adapted to strengthen the work of Germans in Germany; it by no means follows that the same measures or similar measure would strengthen the work of Americans in American surroundings. The chief purpose of this report has been to place the facts before the reader in sufficient detail to permit him to form his own conclusions from the German system. At the same time, it may be permissible to express, in the sequel, some of the opinions which have been formed after personal contact with both systems.

First, a few things which do *not* account for the disparity. It is not to be attributed to greater natural talents or to superior mathematical ability in the average German. It did not appear to me that the raw material for teachers or pupils was better than in America. In fact, as a nation, the United States has already given indications of a marked mathematical *bent;* to lead the nations in inventions, to be noted the world over as quick-witted thinkers (skilled in drawing conclusions, making inferences), to have as a national characteristic the extent to which individuals grapple with and solve practical problems as they present themselves (independent thinking, "individual initiative"), requires the same talents and aptitudes, the same forms of mental activity which lead to success if suitably applied to mathematical problems. Not every mathematician would be a good inventor, because the problems which the latter has to solve are much more complex than those of mathematics, but it can hardly be doubted that every successful inventor would have attained equally good results if he had, with proper guidance, turned his energies into mathematical channels.

The disparity: how not caused.

Nor is the disparity to be attributed to the fact that the German higher schools, which are not free and open to all as are the American

schools, have to some extent "picked pupils." While it is possible that the German regulations may exclude a few pupils whose family antecedents and home surroundings are not conducive to excellence in school-work, yet all the American teachers who have been consulted (including several of wide experience) agree in the opinion that the mathematical work here would not be strengthened if the same regulations were put into effect (*viz.*, a tuition fee of $30 to $40 charged). Some of the best work is often done by pupils whom a tuition fee would debar from the course, and the average of the ability and industry of the classes would not be raised if those were excluded who could not or would not pay a moderate tuition fee.

The causes.

The causes of the excellence of the Prussian work in mathematics may be classed under three heads:

1. *The Central Legislation and Supervision.*
2. *The Preparation and Status of the Teachers.*
3. *The Methods of Instruction.*

Of these, each is in a large measure a consequence of the preceding. Well-prepared teachers are likely to use good methods of instruction and to evolve better ones. Thorough preparation of teachers is sure to be required when the legislative and executive authority is vested in experienced educators of the highest order. We shall consider the three heads in

detail immediately; to summarize, it might almost suffice to say that the strength of the Prussian system may be said to be due to the fact that for two-thirds of a century the work has been centred in a single source of authority for the entire kingdom, whose directions have been carried out systematically and uniformly under the best guidance and supervision the nation could afford.

I. **Central Legislation and Supervision.** — We have seen that the German school system is thoroughly organized on the principle of central authority. The teacher is not amenable to the local community, directly or indirectly. If in any quarter there is dissatisfaction with the work of a teacher, the only recourse is to bring the matter to the notice of his superiors in the educational work. Similarly, within the work, each is amenable only to his superiors, ending with the Minister, who is amenable only to the King.

Each officer, from the Minister to the youngest teacher, is thus free to carry out his share of the work as may seem to him pedagogically best, with the assurance that only his superiors in the work, who have its success equally at heart with himself and who presumably are better qualified to determine its measures and policy, will sit in judgment upon his work.

Expert government.

There can be little doubt that this system secures the greatest efficiency in the work of the German people. A thoroughly military nation, trained for generations to respect for authority, accustomed from infancy to subordination and prompt obedience to superiors, dominated in all business and social relations by the same spirit, the Germans have naturally developed a school system fitting best into their peculiar national temperament and social conditions.

Other conditions.

It is uncertain whether or not the same system would work equally well under other circumstances. In particular, it is hard to determine what degree of centralization is best in the United States. Despite the many considerable advantages of centralized administration, some of the States (*e.g.*, Massachusetts) which leave the administration of school affairs almost or quite wholly in the hands of the local community, have school systems among the best in the country.

A desideratum.

The excellence of the Prussian system certainly emphasizes one desideratum—*viz.*, that whatever may be the extent of the territory subject to one authority, whether a single village or an entire state, the *actual authority*, legislative as well as administrative, be vested in *experienced educators.*

This is the practical way of looking at the

matter and is quite in consonance with American business methods. So long as ultimate authority in educational matters is exercised by men who are not practical educators, who are in fact *theorists* in education, able at most to give some time spared from other business to speculations on educational questions, so long as non-educators have the right to say to the practical educators who are doing the actual work, "You are in our employ; we legislate, you have but to carry out our enactments," for just so long will our educational system stand in the same danger of falling short of the best results which might be achieved that would arise if a body of teachers, be they never so earnest and well-meaning, were to try to manage a great business enterprise in their leisure hours.

Besides the general trend of centralization, a few direct results should be mentioned:

1. *Uniformity of Curricula.*—This is not the place to discuss the desirability of uniform curricula in all the institutions of the same character throughout a state or country. Modern habits of life, however, bring institutions into touch with one another in many ways and make a considerable amount of inter-institutional migration on the part of the pupils inevitable. The work of the pupils who have made a change, as well as that of the classes into which they enter, will

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be more or less retarded if a readjustment of curriculum is necessary. Under the Prussian system, the transfer of pupils from one institution to another causes the minimum of disturbance in the work of either pupils or classes. Practical uniformity of curricula is achieved and maintained better under central authority than in any other way.

2. *The Preparation of the Curricula.* — The task of getting up or modifying the curricula for an entire country is a serious one and may well engage the time and attention of the greatest educators in the country, as in the Berlin conference mentioned above. It is not unwarranted to believe that stronger curricula will be set up in this way than when the curricula are prepared by local talent. Where there is no central authority, advisory bodies, like our Bureau of Education and various voluntary national and State associations, may make recommendations, but their effectiveness is but slight in comparison with the enactments of a legislative authority, and the mode of their preparation must be influenced by the fact that not a *decision* but a plea is being prepared, to which perhaps little attention will be paid by many of those who ought to profit by it most.

3. *Supervision.* — The desirability of competent supervision of the work of instruction is beyond question, and it is nowhere so highly

developed and well organized as under a strong central system. When uniform curricula have been prepared by a central office, careful supervision of the work naturally follows.

4. *American Conditions.*—There are in force in our country to-day only a few *types* of curricula regarded as representing different educational principles or purposes, but there are unnumbered *different* curricula modelled upon one or the other of these types—curricula whose differences among themselves are admittedly of no great educational significance, but which nevertheless require readjustment of work and almost inevitably entail loss upon pupils who are obliged to change from one institution to another. Considerable as this loss is, it is trifling in comparison with that which is due to the fact that during the nine years which correspond to the German gymnasium every one of our pupils *must* make *two* changes of institution—from the Grades to the High-School, and from the High-School to the college. It is unfortunate that with us the work of these nine years is carried on in three different types of institutions, under different management, with different methods and aims, and with teachers differing radically in the character of their preparation. The subject of mathematics is very sensitive to these disturbances and disadvantages, and without doubt

A serious loss.

suffers considerable through them. It is easier to point out the loss than to specify the remedy, but the ill effects of the break between the Grades and the High-School would be reduced and the mathematical work in particular would be much strengthened if the following recommendation of the Committee on College Entrance Requirements of the National Educational Association[1] were put into effect:

"IV. RESOLVED: *That we favor a unified six-year high-school course of study, beginning with the seventh grade.*"

An admirable list of reasons in support of this resolution is given in the report. In the two years to be incorporated in the High-School course, geometry and algebra should be begun and frequently are begun. It would be a great gain if at this point the work in mathematics could be taken up by trained teachers of mathematics and continued without a break for six years under teachers of equal preparation working as members of the corps of instruction of the same institution. If, further, it should be found practicable in the development of the High-Schools to enlarge the course by the addition of a year or two, the mathematics usually required in college would also be included

[1] See p. 30 of Report, submitted July, 1899.

in the extended High-School course, thus collecting under one teaching force the entire course in mathematics (excepting arithmetic) usually required as part of a liberal education. It would seem promising for the work in mathematics if this were done whenever the High-Schools are equipped with well-prepared, enthusiastic teachers of mathematics competent to take up this work. Perhaps the High-School of the future will have a course of eight years instead of four.

Quite a little can be done, moreover, before any organic change in the constitution of the school system, like that recommended by the Committee on College Entrance Requirements, can be achieved. No exception will probably be taken to the statement that it would be a decided advance,

What can be done now.

a. If the teachers and administrators of the three classes of institutions now giving instruction in mathematics were to come into much closer touch and make serious and systematic efforts to bring their work into better correlation; and

b. If all those curricula now in force which are constructed on the same lines, and are already in fact *essentially* the same, were also brought into working uniformity.

It is gratifying to note that much progress is being made along these lines. It is facilitated

by frequent joint conferences and formal or informal organizations of institutions for the purpose of harmonizing their work. If these opportunities are cordially embraced, much can surely thus be done gradually to diminish the tremendous waste which our present system engenders.

II. **The Preparation and Status of the Teachers.**—After what has already been said in the description of the Prussian system, it is almost superfluous to add that the excellence of the Prussian work in mathematics may be attributed to the scientific and pedagogic preparation of the teachers for their work, more largely perhaps than to any other single cause. There must be taken into account, however, not simply the preparation of any one generation, but the cumulative effect of generation after generation of thoroughly prepared teachers. The instruction which the pupil receives is a series of object lessons in teaching, and thus the well-taught pupil has a distinct advantage at the start, whereby the pedagogic methods which one generation evolves are to some extent inherited without effort by the next.

Need of pedagogic training.

But this is only a beginning. The pupil's point of view and that of the teacher are very different. One views his experiences as a pupil from a pedagogic standpoint only through the vista of years.

After long trial the Germans have found that one year's pedagogic preparation in the form of what may be called an apprentice year (the trial year) is insufficient, and have added another year preliminary to this—the seminary year, already described. The Prussian teacher has thus had two years of pedagogic preparation as well as three of scientific training in his specialty when he enters the rank of the teachers. So long as the recruits to our teaching ranks include many with inadequate scientific preparation and with absolutely no pedagogic training, we have no reason for disappointment if the comparison between our work and that done by a carefully trained army of teachers does not result in our favor.

What can be done? Happily much *is* being done. It is a most promising fact that the need for more thorough scientific and pedagogic preparation of our prospective teachers for their profession is widely felt, that the facilities offered for such preparation are constantly being increased, and that the teachers, active and prospective, are availing themselves of these facilities in rapidly growing measure. Only a few decades since, all Americans who wished to study advanced mathematics were obliged to go to foreign lands; now ample opportunities for such study are offered from ocean to ocean in this land and are being eagerly embraced by

increasing numbers of students. With the due co-operation of the appointing authorities, the increasing supply of candidates who have had a more extended mathematical training will of itself raise the standard for the scientific equipment of the teachers of mathematics in our secondary schools, though it will doubtless require a considerable number of years to reach the time when (as in Prussia) a large fraction of the teachers will be Doctors of Philosophy in mathematics, when all will have fulfilled the minimum time-requirement of advanced study for this degree, and when all will have had two years of training in the teaching of mathematics. Until this is the case, some disparity between the German work and our own must be expected.

In the meanwhile, the Prussian results encourage us to push on along the lines which we have already laid down. If there is one thing which the Prussian system teaches clearly, it is that scientific study (the subject-matter) and pedagogic training must both be given great weight in the preparation of the teacher. If either is slighted, the highest efficiency is not likely to be attained.

Raising of standards.

The character of the problems with which mathematics deals, the methods which it uses, and the faculties of the mind to which it appeals are such that the study and

the teaching of mathematics (and hence the preparation for teaching it) will be less affected by differences in national temperament and conditions than the teaching of any other subject. What has proved successful in the preparation of the teachers of mathematics in one nation may be expected also to be successful under equally favorable circumstances in another nation. Still the introduction of any systematic requirement of specific preparation for teachers is too closely connected with the problems of general educational regulation and administration to be handled separately, and present progress with us is likely to be along the line of local raising of the standard for new appointment and voluntary strengthening of their equipment on the part of expectant teachers. In this connection, the third resolution of the Committee on College Entrance Requirements[1] should be mentioned:

"III. RESOLVED: *That the teachers in the secondary schools should be college graduates or have the equivalent of a college education.*"

In addition to the reasons given in support of this resolution in the report, the excellence of the work under the Prussian system may also be adduced. It is decidedly a step in the

[1] Report, p. 30.

right direction which it seems feasible to take now whenever new appointments are made.

Some training of beginners in the teaching of mathematics may likewise be feasible now in institutions where there are one or more teachers with an aptitude for assisting and guiding the beginner in his first teaching. Whether this relation should be formal or informal, and whether the mathematical staff could hold with profit occasional or regular meetings for the discussion of pedagogic questions, are matters for local circumstances to determine. Under favorable conditions quite a little might be done in this way.

Enthusiasm and devotion needed.

However important the scientific and pedagogic preparation of the teacher may be, it will bear its ripest fruit only when applied with devotion and enthusiasm throughout a lifetime. To this, the *status* of the Prussian teachers is especially conducive. To the German teacher his profession is a life-work, and not a stepping-stone or adjunct to something else. Once fairly launched upon it, he almost never leaves it voluntarily, and his tenure of his position is assured for life, barring his own gross fault. He does not read law or practise medicine when free from the school duties which "keep the pot boiling" until he is prepared to abandon the teacher's vocation for one more lucrative or to his taste. On the con-

trary, he is fully alive to the fact that his teaching is a life-work, and that its honors and prizes will come mainly in the later portion. The admirable salary and pension system frees him from harassing financial care and worry, and thus, with fully determined and assured official position and income (promotions coming in due course according to fixed regulations), he can give his entire thought and energy to the actual work of his profession.

The result is that the young man is not in haste to realize on his few professional assets in his youth, before they shall have lost value to to him through his passing on to some more desirable position or occupation, but he steadily lays a deep and solid foundation for many years of future work. The traditions of the profession, his relations with his colleagues, far and near, pedagogic meetings and conferences, the constant stream of publications which is flowing from these schools, all contribute to keeping enthusiasm and ambition alive. We Americans are not likely to overestimate the effect which the *status* of the Prussian teacher has upon the thoroughness and efficiency of his work.

III. **The Methods of Instruction.**—The following points may be mentioned as deserving special attention under this head:

1. *The Stress Laid on Classroom Work.*—By far the larger part of the work as to quantity and

all of the work of acquisition, *all* of the advance, *all* of the study of critical points, are done in the class-exercise, under the direct guidance and stimulus of the teacher. The German pupil receives from twenty-five to thirty hours per week of actual instruction, and his home-work (almost entirely of a drill or routine character) ranges from six to eighteen hours per week according to age.

Work under instruction.

The advantage which the pupil gains from doing so large a fraction of his work in the class-exercise is of course dependent upon the teacher; the better the teacher, the more the pupil will gain from his instruction. The better the teacher, the better also he can utilize the time at his disposal. When the less well-prepared teacher must in the smaller fraction of time give such instruction as will enable the pupil to use the larger time of private study to best advantage, the results cannot but fall short of the best.

One consequence of the quantity and character of the private work which the American pupil does is that he must receive a large portion of his instruction from a text-book. Whether this is desirable or not need not be discussed here; it is a fact that even with the best of teaching the pupil needs some guidance in his private work, and so long as so large a fraction of the American pupil's work *is* private

work a good text-book would seem to be the best substitute for the living teacher.

In this connection the following recommendation of the Committee on College Entrance Requirements should be noted:[1]

"XIV. RESOLVED: *That we recommend an increase in the school day in the secondary schools, to permit a larger amount of study in school under supervision.*"

Among the remarks in support of this resolution are the following:

"In presenting this resolution the committee is aware that there is a great divergence in the length of the day in the secondary schools, the number and length of recitation periods, the noon intermissions, and the time devoted sacredly to study within the school-house. A few have two sessions, following the rule governing the elementary schools; some are from 8 A.M. to 1 P.M., and many from 9 A.M. to 2 P.M., with one-half hour a noon for a light lunch.

We appreciate the almost unanimous and perhaps enlightened opposition on the part of teachers to the proposition for a longer school day. The committee believes, however, that it is a subject for intelligent discussion, and that

[1] Report, p. 40.

the weight of argument favors a longer day. The committee does not trace its convictions on this matter to the fact that the German secondary schools are one-half longer in session than our schools and no hardship seems to result.

There is no disposition to imitate European methods because they are European, but we believe it is easily demonstrable that it is in the class recitation and under the inspiration and instruction of the teacher, and not in study hours at home, that the pupil acquires the bulk of his scholastic knowledge."

Though the Committee justly bases its recommendations upon broader ground than the fact that German schools have longer sessions than our schools, the scope of the present report is restricted precisely to a study of the German schools, and while the teaching of the German schools is along the same general line, it differs in some details from the recommendations of the Committee.

The important point is not so much that the German school day is longer (from 8 A.M. to 1 P.M. is quite customary in Berlin), as that the *time of instruction* is longer for each pupil. Such a thing as "study in school under supervision" is unknown in the Prussian schools. The pupil is participating in class-work, receiving instruc-

More instruction needed.

tion during the entire time he is at school. The hour-schedule is invariably arranged so that all the work of each class is consecutive, and the class is dismissed as soon as its instruction is ended. Neither pupils nor teachers are expected to be present except when receiving or giving instruction.

While doubtless the private study of the pupils will as a rule be more effective if done "in the midst of all the paraphernalia incident to study," actual instruction amid the same surroundings (with a corresponding diminution of the amount of study to be done in private) would be still more effective. To *transfer* study hours from the home to the school would be a gain, but to *transform* study hours into hours of instruction would be a far greater gain. This transformation would hardly arouse much opposition on the part of the teachers provided it did not entail additional burdens upon them.

So far as Prussia is concerned, she does not secure the greater quantity of instruction by requiring her teachers to teach more hours, but by providing more teachers. The Prussian teacher teaches from twenty to twenty-two hours per week with a recess of ten minutes deducted from each hour. This will not compare very unfavorably with the amount of teaching done by American teachers in the corresponding work.

2. *The Pause.*—Of what may be called the minor features of German instruction, none perhaps is more productive of good results than the ten-minute pause at the close of each hour, bringing, as it does, complete relaxation for both pupils and teachers. It is not needful to dwell upon it further than this mention. The American custom of passing from one subject to the next as quickly as the change can be effected, in the attempt to keep the minds of teachers and pupils on a tense strain for several hours at a time, is much to be deplored. It is not true economy to strive to have as few seconds as possible "lost" in the change from one subject of instruction to the next.

It may not be amiss to add that of the thousands of German boys whom I saw during the pause not one was engaged in private study; I saw no hint of a feeling that these minutes might be used to put the last touches to the preparation for the work of the next hour.

3. *The Distribution of the Mathematical Work.*—The three serious differences between the German and the American distribution of the mathematical work—*viz.*, the scope of arithmetic, the order in which the subjects are begun, and the rate at which they are continued—are no longer open questions in Germany. Experience has settled them beyond a peradventure. Arithmetic is restricted

Three points of difference.

to the most essential work of computation, demonstrative geometry is begun before algebra, and the study of each mathematical subject extends over several years instead of being studied at high pressure during a much shorter time, as is done with us. The following table gives the number of years over which the study of each subject extends both in the German schools and in the American schools which have been brought into comparison with them above. In the American schools the subjects are kept quite distinct, and only one subject is taken up at a time. In the German schools the subject of study is *mathematics*, and its various subjects are developed side by side. At no time is only one subject being studied with the exception of the two lowest years (Sexta and Quinta), in which only arithmetic is taken up.

Subject.	Years in Prussia.	Years in America.
Arithmetic	3	4½
Plane Geometry	5	1
Algebra	6	2⅓
Solid Geometry	3	½
Plane Trigonometry	2	⅓

The consensus of American mathematico-pedagogic thought is strongly in favor of reducing the material taken up in arithmetic both by deferring topics to appropriate points

in algebra and geometry and by omitting others altogether. Reforms along this line have already been made and others may confidently be expected. The time thus gained permits the earlier beginning of algebra and geometry. The door is also open for a change in the *order* of beginning these subjects, and for a change from the practice of forcing pupils to bolt them in a mass. Whether these changes are wise and feasible is another question. They certainly seem wise from the stand-point of pedagogic theory. No one, probably, denies that the beginnings of demonstrative geometry are more simple and concrete than those of algebra; unless, indeed, the latter is taught simply as mechanical routine, blind manipulation of symbols according to arbitrary rules, a thing above all others to be avoided, and which even without any encouragement is peculiarly tempting to many beginners in algebra.

Gradual growth.

It is likewise clear that the elements of algebra are simpler than the more difficult parts of geometry, that each of these subjects can be of assistance to the other, and that a ripe understanding of them both is best attained if the work extends over a number of years with gradual progress to the more difficult portions as the pupil's mind develops. If the young pupil with unformed and growing powers is to make progress with understanding

in mathematics, his acquisitions must have time to take root. The *facts* of algebra and geometry may perhaps be acquired equally well in a more concentrated course; the pupil may be equally well prepared for an examination by taking up the subject relatively late and covering it at a more rapid rate, but his grasp will hardly be as firm or as lasting as though he and the subject had grown up together.

So much from the stand-point of pedagogic theory. What can be said from the *practical* stand-point? It is here that the German schools with their thorough trial of the plans just discussed offer a reply of great value. They have tried the experiment for us. The question is practically the same there as here; differences of national temperament and conditions are not of serious consequence; the determination of these questions depends upon far more fundamental and characteristic traits of the mind, common to all nations. While in no sense accepting the German stamp as guarantee of excellence, it would be equally foolish to go to the other extreme and refuse to consider solutions which the Germans have worked out for the same problems with which we are now grappling. The German higher-school system speaks with no uncertain tone of the practical advantages of beginning both geometry and algebra early, the former first, and continuing

their study simultaneously throughout quite a number of years. With the pedagogic theory thus corroborated by practical experience, it would hardly be an experiment to interchange the order of algebra and geometry, and to carry on the subject simultaneously through a longer period. The practical difficulties in the way of simply interchanging the subjects are not very serious, but it would require considerable readjustment of programmes to put the second change into effect. Still the advantage to be gained is sufficiently considerable to warrant strenuous efforts to overcome the difficulties which may be in the way.

4. *The Method of Instruction in Mathematics.* —The manner of instruction in Prussia has already been described. It may be styled the *genetic method.* The subject grows up in the class treatment, which does not require any previous preparation of the pupil for the work of the hour. A textbook may quite well be used with this method of instruction, but the pupil refers to it *after* the class treatment, not before.

The genetic method.

The genetic method has been little used in America, but there is a strong trend toward the use of a somewhat similar method styled the *heuristic method.* This method has been used hitherto almost exclusively in the teaching of geometry. It resembles the ge-

netic method in the marked effort it makes to keep the pupil thinking for himself. It differs from the genetic method in that the class is the working unit in the genetic method while the *pupil* is the working unit in the heuristic method.

This difference may almost be said to be characteristic of the German and the American instruction in mathematics. In Germany the class works as a whole under the guidance of the instructor; when a pupil speaks it is part of a concerted action, as momentary spokesman for the class, as it were. In the American class the individual pupil demonstrates, explains, asks and answers questions; the others listen to him. Whatever the style of instruction, the individual pupil is prominent. It is difficult to give a clear description of the distinction, but it is readily felt by one who visits classes systematically in both nations, and the feeling grows stronger with the number and frequency of the visits. In this desire to give free play to the ability of individuals the difference between the national temperament of the two peoples is quite apparent, and while we cannot cherish this desire too highly nor encourage too warmly every attempt to avoid hampering the growth of the individual, it must not be forgotten that the aim of the class-work is the greatest good of the

Class *vs.* Individual.

greatest number, and that perhaps this may not be attained by giving the pupils in turn individual prominence.

The heuristic method does not permit the use of a text-book of the ordinary style; either a specially prepared text is used or none. In the class-hour the pupil presents what he has already worked out, and the instructor helps him over the difficulties which he has not been able to surmount by himself. The genetic method is firmly established in Germany, and the heuristic method has been used with success in this country, where it has enthusiastic advocates. In either method a text if used plays a very minor part, and success depends upon the aptitude of the teacher to use the method. Teachers should not venture upon either of these methods without special training for their use. The best training that can be suggested for teachers already at work, who feel the impulse toward the heuristic method, often called "teaching without a text," is to seek to digest the subject in contradistinction from its presentation in any particular text, so that, whatever text may be used, it is the *subject* that is being taught. The time for the adoption of the heuristic method is when the teacher feels cramped by any other. The primary desideratum is that the teacher have a thorough mastery of the subject, and this

The heuristic method.

requires that his mathematical horizon be much wider than what is actually taught. No teacher is ready to teach without a text who is not distinctly conscious of teaching the subject even though teaching with a text. When this stage is reached, the use of a text is a matter of comparatively little moment which the teacher's preferences and local conditions will determine. In general, as long as the pupils work apart from the instructor so much of their time, as is the case with us, a good text may often be the best substitute available for the personal guidance of the teacher.

It is not necessary to discuss the classroom methods of the Germans further here. They have been described in sufficient detail to enable each reader to appreciate whatever they may contain that will be helpful or suggestive to him; what would be novel to one will be in daily use in the classroom of another. Our American teachers are eclectic, and have the freedom of a wide range of method; the good features of the Prussian classroom work may be seen, some here, some there, in many an American classroom to-day. It may therefore well be left to each reader to draw what specific conclusions he may think best. If, however, the tale should end with a moral, that moral would undoubtedly be this:

The chief thing is that every teacher be active

and growing scientifically and pedagogically; that he add to his mathematical attainments year by year, and that he study to improve in the art of teaching.

When this is the case throughout our land, all other needed developments will follow.

Finally, the disparity which we have been considering seems to be fully accounted for by the less favorable circumstances under which the American work is done; in view of all the circumstances, we may well be gratified with what we have done and are doing, and I am confident that thoughtful consideration of the situation here and elsewhere will justify the assured expectation that, with the attainment of equally favorable conditions in the various respects which have been discussed, the work done in American school mathematics need not fear the test of comparison.

As this report is on the point of receiving its closing words, the article of Professor Münsterberg, of Harvard, on the German school-work (*Atlantic Monthly*, May, 1900) comes to hand. Professor Münsterberg writes from the point of view of one educated under the German system and learning to know the American system later in life. The present report is written from the point of view of one educated under the American system and learning to know the German system later in life. There is always

danger that a visitor will see the good points of a system, that its differences from the system under which he has been bred and whose shortcomings he knows intimately, will impress him in the most favorable light possible, while the weak points of the system will probably be of a negative and less striking character and be appreciated fully only upon a much closer acquaintance with the system than a visitor ever gets. With this danger in mind, the writer has taken pains to exclude from this report mere opinions and impressions which could not be substantiated by facts and figures, or supported by good reasons. It is an additional gratification to find that Professor Münsterberg finds himself able, after completing the course of a gymnasium, to make even stronger statements as to the excellence of the work done in these institutions than any contained in this report. The words of advice which he gives to American educators are especially pertinent at the present time and deserve most careful consideration.

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